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Journal of the AMERICAN STATISTICAL ASSOCIATION

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VOLUME 41 NUMBER 234

American Statistical Association

Organized November 27, 1839
Incorporated 1841

The American Statistical Association is a scientific and educational organization. Its membership is not confined to professional statisticians but includes economists, business executives, research directors, government officials, university professors, and other persons who are seriously interested in the application of statistical methods to practical problems, in the development of more useful methods, and in the improvement of basic statistical data. Engineers, mathematicians, biologists, actuaries, sociologists, psychologists, and representatives of many other professions are included in the membership of the Association.

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JOURNAL OF THE AMERICAN STATISTICAL ASSOCIATION

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THE MEMBERSHIP OF THE AMERICAN STATISTICAL ASSOCIATION—AN ANALYSIS

ABNER HURWITZ AND FLOYD C. MANN
Bureau of Labor Statistics

IF MR. "X" were a composite of the ASA membership, he would most likely be living in Washington, D. C. or New York City, employed by a government agency as an economist or statistician specializing either in planning and reviewing surveys or collecting or directing the collection of statistical data. His principal field of interest in statistical data would center around economics and economic theory, and if he belonged to another professional society it would probably be the American Economics Association He would have received an advanced degree in one of the social sciences or its disciplines, and would at some time have published an article or a book.

These are the predominant characteristics of the members of the American Statistical Association as reported on a special directory form, which was distributed to all members who could be reached during the early part of 1945, and the information furnished in support of membership applications by those persons joining the Association between January and September 1945. The 1945 questionnaire requested more information about each member than had been obtained in earlier directory forms, and this article, therefore, presents a more comprehensive tabular analysis[1] of the characteristics and interests of the membership than has been available previously.[2] Data were obtained on present occupation and organization by which employed, positions

[1] The detailed tabular analyses presented in this article were made possible through the cooperation of Joseph F. Drager, Chief of the Machine Tabulation Division of the Bureau of Labor Statistics.

[2] For earlier analyses of the composition of the membership of the American Statistical Association, see Willford I. King: "Classification of Members of American Statistical Association on Basis of Duties and Interests," this JOURNAL, Vol. 22, June 1927, pp. 224–226; Stuart A. Rice and Morris Green: "Interlocking Memberships of Social Science Societies," this JOURNAL, Vol. 24, September 1929, pp. 303–306, and "Composition of the American Statistical Association," this JOURNAL, Vol. 25, June 1930, pp. 198–202; and Richard L. Funkhouser: "Membership of the American Statistical Association on its Hundredth Anniversary," this JOURNAL, Vol. 36, September 1941, pp. 329–341.

held since 1940, education, major and minor statistical specialties, fields of interest in statistical data and methods, membership in other professional societies, and chapter and national offices held. Additional occupational data were reported by those members on leave from a company, university, or other organization.

More than 3300 persons were listed in the 1945 Membership Directory but the largest number of persons reporting information for any one question was 2942. The following tables are based on the total number of members furnishing information in answer to each question.

DISTRIBUTION OF MEMBERSHIP BY TYPES OF EMPLOYING ORGANIZATIONS

Almost 35 per cent of the members reporting their employing organization in 1945 were employed by government agencies, one-sixth were with colleges and universities, and one in ten was associated with a manufacturing or industrial company. Twelve per cent were in the military services. Table 1 shows the types of organizations with which

TABLE 1

TYPE OF ORGANIZATION BY WHICH MEMBERS OF THE AMERICAN STATISTICAL ASSOCIATION WERE EMPLOYED IN 1940 AND 1945

Type of Organization	Members Reporting Employment			
	1945			1940*
	Number	Per Cent	Per Cent Excluding Military	Per Cent
All A.S.A. members reporting:	2942	100.0	100.0	100.0
Government agencies:	1019	34.6	39.4	38.0
Federal	862	29.3	33.3	29.1
State	130	4.4	5.0	7.3
Local	27	0.9	1.1	1.6
Colleges and universities	492	16.7	19.0	23.4
Financial institutions	202	6.9	7.8	10.4
Public utilities, transportation, or communications companies	115	3.9	4.5	3.9
Manufacturing industries	324	11.0	12.5	5.2
Retail and wholesale trades	21	0.7	0.8	1.5
Other organizations:	373	12.7	14.4	16.0
Business research	143	4.9		
Trade associations	62	2.1		
Research foundations	60	2.0		
Charitable organizations	36	1.2		
Other	72	2.5		
Military services	354	12.0		
Retired	26	0.9	1.0	1.6
None	16	0.6	0.6	—

* Source: Richard L. Funkhouser, *op. cit.*, p. 330.

members reported employment in 1945 and provides a comparison with the membership's employment connections in 1940.[3]

A comparison of the 1940 and 1945 organizational distributions provides some evidence of how employment of the membership of the American Statistical Association was affected by World War II. In spite of the withdrawal of members into the military services, the per cent of members employed by the Federal Government, public utility, transportation and communications companies remained unchanged and the per cent of members in manufacturing industries increased two-fold. The number of members connected with colleges and universities, financial institutions, trades, and other organizations decreased considerably.

This resulted from members shifting into government and industry from universities and other non-war activities, and the attraction of new members into the Association from the greatly increased Federal payroll and from industrial plants in which the widespread introduction of quality control techniques attracted many users of statistical methods.[4] These comparisons are more striking when percentages for 1945 are computed excluding members in military services. Of the civilian members who reported the organizations by which they were employed, one-third were working in Federal agencies and one-eighth were employed in manufacturing.

[3] The above distributions are not strictly comparable with that prepared under the direction of W. I. King in 1927. However, certain groupings can be made to provide a rough measure of the per cent of members employed by different types of organizations since 1926.

Combinations of Organizations	Per Cent		
	1926	1940	1945
Government agencies, colleges and universities (including federal, state and municipally employed)	42.8	61.4	51.3
Financial institutions (including insurance)	15.0	10.4	6.9
Public service corporations	8.4	3.9	3.9
Manufacturing (including publishers and printing)	12.6	5.2	11.0
Mercantile	3.0	1.5	0.7
Professional service organizations (including business research, charitable and other organizations)	8.4	16.0	8.6
Trade associations	4.1	—	2.1
Foundations, etc.	5.7	—	2.0
Retired	—	1.6	0.9
Military	—	—	12.0
None	—	—	0.6
Total	100.0	100.0	100.0

[4] The increasing use of statistical controls in industry is evidenced by the organizational affiliations of the more than 400 members elected to the Association between September 1945 and May 1946. More than 25 per cent of these new members are employed in manufacturing plants as statisticians, quality control engineers, chemists and metallurgists, and another 11 per cent are connected with industry in other capacities.

OCCUPATIONAL DISTRIBUTION OF MEMBERSHIP

Nearly 29 per cent of the members reporting occupational information identified themselves as economists, 27 per cent had the rank or title of statistician, and almost one-eighth were executives, directors, or managers. The remaining third reported a wide range of occupations,

TABLE 2

OCCUPATIONS OF MEMBERS OF THE AMERICAN STATISTICAL ASSOCIATION IN 1945

Occupation	Members Reporting Occupation			
	Number		Per Cent	
All A.S.A. members reporting:		2550		100.0
Economists in the fields of:		730		28.6
Prices, production, etc.	117		4.6	
Labor and employment	84		3.3	
Finances, taxes, and insurance	93		3.6	
Marketing and trades	96		3.8	
Agriculture	46		1.8	
Transportation, communication, and public utilities	27		1.0	
Social science	24		1.0	
Other research	111		4.3	
Teaching	132		5.2	
Statisticians in the field of:		684		26.8
Economics	172		6.7	
Social sciences	146		5.8	
Physical and biological sciences	22		0.9	
Mathematics	27		1.1	
Business and management	62		2.4	
Industry	70		2.7	
Other research	107		4.2	
Teaching	78		3.0	
Executives, directors, and managers		304		11.9
Other administrative officers		49		1.9
Auditors and accountants		69		2.7
Marketing specialists		87		3.4
Mathematicians and actuaries		101		4.0
Engineers, industrial		28		1.1
Engineers, other		25		1.0
Psychologists		48		1.9
Sociologists		38		1.5
Social scientists		35		1.4
Physicians and public health officers		33		1.3
Journalists		24		1.0
Chemists		24		1.0
Biologists		18		0.7
Botanists		19		0.7
Other professions		88		3.4
Research directors		31		1.2
Research assistants		50		2.0
Clerical		46		1.8
Students		19		0.7

including mathematicians, marketing specialists, sociologists, and physical scientists. The heavy occupational representation of the social science fields in the Association is shown in Table 2.

The information in the above table was obtained from the first section of the 1945 Directory questionnaire, which asked for data on each member's "rank or title" and "description of duties or functions." The problems encountered in attempting to classify this type of data into mutually exclusive occupational categories were formidable, and the classifications used provide only a rough index to functions and job responsibilities of the members. Persons who had titles as chiefs of government agency units but whose duties, according to the descriptions provided, were primarily those of an economist, social scientist, etc., were classified according to the description. No person was classified as a statistician unless his job title or the description of his duties indicated definitely that he should be so classified. War-time security restrictions precluded many members in the military services from furnishing specific job descriptions. These members were, therefore, not included in this summary.

Since more than half of the Association's members identified themselves as statisticians and economists, these two groupings were classified further according to the principal field in which the member's job responsibility was centered. Forty-six per cent of the "statisticians" were working in economics or one of the other social science fields; 18 per cent of the economists were teaching, one in six was working in the fields of prices and production, and 13 per cent in marketing or the trades.

In addition to the more than 8 per cent of the membership who were teaching economics and statistics, another 8 per cent were engaged in teaching in other fields, the most important of which were mathematics, business administration, and psychology. Members teaching subjects other than economics and statistics are not shown separately in Table 2, but were included under their major occupational group. Students in September 1945 represented less than one per cent of the membership.[5]

OCCUPATIONAL DISTRIBUTION BY EMPLOYING ORGANIZATION

Table 3 shows the occupations of the members reporting this information by the type of organization in which they were employed in 1945. More than one-half of the statisticians in the Association were engaged in government work, 14 per cent were employed by universities

[5] Between September 1945 and May 1946 the number of student members more than doubled.

TABLE 3

OCCUPATIONS OF AMERICAN STATISTICAL ASSOCIATION MEMBERS BY EMPLOYING ORGANIZATION IN 1945

Occupations in 1945	Members Reporting Employment in										Members in Military Services in 1945[1]	
	All Organizations		Government Agencies	Universities and Colleges	Financial Institutions	Public Utilities, etc.	Manufacturing and Trades	Business Research and Trade Ass'ns.	Other Organizations	Organization not Reported		
	Number	Per Cent									Number	Per Cent
All A.S.A. members reporting—Number	2586	—	1019	492	202	115	345	205	168	40	354	100.0
Per cent	100.0	—	39.4	19.0	7.8	4.5	13.3	7.9	6.5	1.6	—	—
			(Per cent of members reporting)									
Statisticians	684	100.0	51.8	14.3	3.2	6.0	12.1	5.7	6.3	0.6	71	20.0
Economists	730	100.0	54.7	19.4	5.9	3.0	6.0	5.9	4.0	1.1	78	22.0
Executives, directors and managers	304	100.0	17.7	6.6	26.0	5.9	19.4	14.5	6.6	3.3	20	5.7
Other administrative officers	49	100.0	44.9	0.0	4.1	10.2	26.5	4.1	10.2	0.0	7	2.0
Accountants and auditors	69	100.0	18.8	2.9	13.0	11.6	34.9	11.6	5.8	1.4	6	1.7
Marketing specialists	87	100.0	2.3	1.1	18.4	2.3	33.4	40.2	2.3	0.0	10	2.8
Engineers (including quality control engineers)	53	100.0	5.7	7.5	0.0	15.1	60.4	7.5	3.8	0.0	7	2.0
Mathematicians and actuaries	101	100.0	17.8	53.4	19.8	0.0	2.0	4.0	0.0	3.0	8	2.3
Psychologists, sociologists and social scientists	121	100.0	32.3	45.4	0.0	0.0	3.3	3.3	14.9	0.8	17	4.7
Biologists, botanists and chemists	61	100.0	27.9	27.9	0.0	0.0	29.4	3.3	11.5	0.0	3	0.8
Research directors and assistants	81	100.0	32.2	14.8	4.9	1.2	11.1	11.1	24.7	0.0	7	2.0
Other professionals	145	100.0	20.7	45.5	0.7	1.4	9.0	4.1	9.6	9.0	14	4.0
Clerks	46	100.0	49.9	2.2	8.7	8.7	10.9	10.9	8.7	0.0	9	2.6
Students	19	100.0	0.0	100.0	0.0	0.0	0.0	0.0	0.0	0.0	2	0.6
Occupations not reported	36	100.0	52.9	2.8	5.5	11.0	27.8	0.0	0.0	0.0	95	26.8
Members in military services in 1945*												
Number	354	—	136	44	16	14	27	1	26	90	—	—
Per cent	100.0	—	38.4	12.4	4.5	4.0	7.6	0.3	7.4	25.4	—	—

* Occupations and organizations in which members were employed before entering services.

and colleges, chiefly as teachers, and 12 per cent by manufacturing and business organizations. Fifty-five per cent of the economists were with the government and almost half of these were working in the fields of prices, production, labor, and employment. About 75 per cent of the executives, directors, and managers were connected with business organizations.

One-fourth of the members in the military in 1945 did not report their occupations or the type of organization with which they were associated prior to entering the services. However, more than half of those reporting had been employed as government statisticians and economists. Sixteen per cent had been connected with universities and colleges.

TYPES OF STATISTICAL WORK PERFORMED

Slightly more than 50 per cent of the members of the American Statistical Association indicated that they had special skills in the planning and review of plans for surveys and in the collection or the direction of the collection of statistical data. About 48 per cent considered themselves to be skillful users of statistical data and to be proficient in sampling and analyzing data. Nearly twice as many indicated special abilities for general research as for teaching.

Although more than half of the membership specialized in planning and reviewing of plans for surveys, less than one out of every five considered this to be his major specialty. Sampling and the analysis of statistical data was checked by 15 per cent as a major skill.

The relationship between the types of statistical work regarded by the membership as major specialties and the types checked as other special skills is shown in Table 4. The different types of statistical work have been ranked according to the frequency with which each was checked either as a major or an "other" special skill.

On the average the different types of statistical work were checked as "other special skills" 2.6 times as frequently as they were indicated as "major specialties." It is not surprising that the more specialized types of work—classification; graphic presentation; index number construction; editing, coding, or tabulating—were less frequently designated as major specialties and more frequently checked as "other special skills." More than 7 times as many considered graphic presentation as a minor skill as compared with the number of members who designated it as their major specialty.

Space does not permit the presentation of an analysis in this article of the major patterns of statistical work in which the members reported

* Occupations and organizations in which members were employed before entering services.

TABLE 4

SPECIAL SKILLS IN STATISTICAL WORK PERFORMED BY AMERICAN STATISTICAL ASSOCIATION MEMBERS IN 1945*

Type of Statistical Work	Members Reporting Statistical Work Performed as					
	Total		Major Specialty		Other Special Skill	
	Number	Per Cent	Number	Per Cent	Number	Per Cent
All A.S.A. members reporting:	2715	100.0	—	—	—	—
Planning and review of plans for surveys	1372	50.5	510	18.8	862	31.7
Collecting or directing the collection of data	1368	50.4	355	13.1	1013	37.3
User of statistical data	1304	48.0	323	11.9	981	36.1
Sampling and analysis	1300	47.9	416	15.3	884	32.6
General research	1189	43.8	351	12.9	838	30.9
Editing, coding, and tabulating	850	31.3	154	5.7	696	25.6
Graphic presentation	809	29.8	95	3.5	714	26.3
Teaching	622	22.9	266	9.8	356	13.1
Index number construction	346	12.7	50	1.8	296	10.9
Quality control	309	11.4	74	2.7	235	8.7
Classification	182	6.7	15	0.6	167	6.1
Actuarial methods	151	5.6	63	2.3	88	3.3
Other statistical work	32	1.2	4	0.2	28	1.0

* 528 members who reported type of statistical work performed did not indicate any skill as their major specialty. 436 members indicated two or more types of work as their major skill, and thus there is duplication in these figures.

specialization. However, survey planning, data collection, sampling, and editing, coding and tabulation appear most often in combination.

FIELDS OF INTEREST IN STATISTICAL DATA AND METHODS

A larger number of the members indicated interests in statistical data and methods in the fields of economics and economic theory than in any other fields. Prices and production was the subject most frequently designated as of primary importance. One-third of the Association checked this field as an area in which their interests centered. One in four reported interest in labor and employment economics, 20 per cent noted population research as an interest, and more than one-seventh rated the following as areas of primary importance: domestic and international trade, marketing and advertising, sociology and social service, vital statistics and public health, and mathematical sciences.

As would be expected, the number of members interested in different fields varies considerably when these areas of concentration are classified as of primary, secondary, or minor importance. While 12 per cent indicated population research to be a field in which their interest in

statistical data and methods centered, less than 3 per cent ranked it as an interest of primary importance. Similarly, although only one per cent rated opinion polling as their first interest, 8 per cent reported it as an interest field.

The first six fields which the membership ranked as of primary importance were: prices and production, labor and employment, general economics and theory, marketing and advertising, vital statistics and

TABLE 5

FIELDS OF INTEREST IN STATISTICAL DATA AND METHODS OF MEMBERS OF THE AMERICAN STATISTICAL ASSOCIATION IN 1945

Fields of Interest in Statistical Data and Methods	All Members Reporting Interest		Members Ranking Interests as of:					
			Primary Importance		Secondary Importance		Minor Importance	
	Number	Per Cent	Number	Per Cent	Number	Per Cent	Number	Per Cent
All A.S.A. members reporting:	*2766	100.0	—	—	—	—	—	—
Economics and economic theory:								
General interest	460	16.6	182	6.6	213	7.7	65	2.3
Prices and production	934	33.8	384	13.9	336	12.2	214	7.7
Labor and employment	650	23.5	209	7.6	173	6.2	268	9.7
Domestic and international trade	415	15.0	55	2.0	148	5.3	212	7.7
Public finance and taxation	354	12.8	78	2.8	91	3.3	185	6.7
Transportation and public utilities	244	8.8	86	3.1	57	2.1	101	3.6
Business and management:								
General interest	338	12.2	109	3.9	100	3.6	129	4.7
Marketing and advertising	421	15.2	134	4.8	126	4.6	161	5.8
Accounting and private finance	339	12.3	108	3.9	100	3.6	131	4.8
Personnel	268	9.7	34	1.2	88	3.2	146	5.3
Production design and prod. control	229	8.3	62	2.2	63	2.3	104	3.8
Population research	547	19.8	68	2.5	148	5.3	331	12.0
Sociology and social service	415	15.0	125	4.5	84	3.0	206	7.5
Vital statistics and public health	408	14.8	126	4.6	91	3.3	191	6.9
Mathematical sciences	382	13.8	104	3.8	84	3.0	194	7.0
Opinion polling	311	11.2	34	1.2	67	2.4	210	7.6
Agricultural economics and rural statistics	297	10.7	89	3.2	53	1.9	155	5.6
Psychology	252	9.1	60	2.2	52	1.9	140	5.0
Education	243	8.8	56	2.0	60	2.2	127	4.6
Biological sciences	237	8.6	59	2.1	68	2.5	110	4.0
Physical sciences and engineering	200	7.3	54	2.0	54	2.0	92	3.3
Medicine	199	7.2	41	1.5	82	3.0	76	2.7
Agricultural experimentation and production	143	5.2	41	1.5	38	1.4	64	2.3
Chemical sciences	75	2.7	10	0.4	18	0.6	47	1.7
Other fields of interest	61	2.2	38	1.4	11	0.4	12	0.4

* 420 members reported but did not indicate the order of importance of their interests. The primary interests of these members were determined by observation of related information (e.g. occupation, major field of education) but ranked as secondary since no major importance was actually indicated.

public health, and sociology and social service. Table 5 shows how the members rated their different fields of interest in statistical data and methods according to importance.

Of those members ranking prices and production as their field of chief interest, 23 per cent indicated that domestic and international trade was their second area of interest, and about the same number checked labor and employment as of minor importance. The predominant interest pattern of those ranking marketing and advertising as primarily important showed prices and production and opinion polling as second and third interests, respectively. Other predominant patterns of interests are as follows:

Product design and production control, prices and production, and physical sciences and engineering.
Sociology and social service, population research and census, and vital statistics and public health.
Education and psychology.
Agricultural experimentation and biological sciences.
Physical sciences, engineering, and mathematical sciences.

AFFILIATION OF MEMBERSHIP WITH OTHER PROFESSIONAL SOCIETIES

Almost 34 per cent of the American Statistical Association members answering the question concerning memberships in other professional societies indicated that they had no other affiliations, 28 per cent belonged to one other society, 17 per cent were affiliated with two, and the remaining 21 per cent held memberships in three or more other professional organizations.

Two out of every five members who were affiliated with other groups belonged to the American Economic Association; almost 17 per cent were members of the Institute of Mathematical Statistics. In Table 7 the societies which were listed on the directory form have been grouped according to subject matter; the unlisted societies entered on the directory form by members have also been included in these general categories. Since 38 per cent of the membership reporting indicated that they belonged to two or more other societies, extensive duplications exist in the first two columns of this table.

More than one-half of the members belonging to at least one other society indicated that they were affiliated with an economics group, more than one-third were affiliated with one or more of the "political and social science" societies, and almost 23 per cent were members of other associations. More than one out of every five members be-

longed to a mathematical or other statistical organization. Eleven per cent were associated with one of the societies classified under business management, and six per cent were members of psychological and educational associations. Although the members of the American Statistical Association were for the most part interested in the social science fields, it is notable that almost 13 per cent of the membership belonging to other professional societies were members of physical and biological science associations, and almost 15 per cent belong to the American Association for the Advancement of Science, an organization composed of persons having a diversity of interests in the sciences.

Some measure of the diversity of interests of the individual American

TABLE 6

NUMBER OF OTHER PROFESSIONAL SOCIETIES TO WHICH AMERICAN STATISTICAL ASSOCIATION MEMBERS BELONGED IN 1945

Number of Professional Societies with which Affiliated	A.S.A. Members	
	Number	Per Cent
All A.S.A. members reporting:	2796	100.0
None other	942	33.7
One or more other	1854	66.3
One other	787	28.2
Two others	479	17.1
Three others	306	10.9
Four others	148	5.3
Five others	59	2.1
Six others	45	1.6
Seven others	12	0.4
Eight others	7	0.3
Nine or more others	11	0.4

Statistical Association members is provided by the data in Table 7 which show those members who belonged to one other society or to only societies classified under one major subject matter group. Fifty-eight per cent of the members who belonged to other societies were associated with only one of these major groups, about 27 per cent belonged to one or more economics associations, and 12 per cent held memberships in "political and social science" societies. Although more than 22 per cent of the American Statistical Association members who belonged to other societies were members of some mathematical or other statistical association, only 10 per cent belonged exclusively to societies in this group.

Diversity of interests of individual American Statistical Association members is apparent from figures which show that almost 42 per cent

TABLE 7

AMERICAN STATISTICAL ASSOCIATION MEMBERS' AFFILIATIONS WITH OTHER PROFESSIONAL SOCIETIES IN 1945

Professional Societies and Groups	A. S. A. Members					
	Belonging to One or More Other Professional Societies		Belonging to Only One Other Professional Society		Belonging to Societies in Only One Group*	
	Number	*Per Cent*	*Number*	*Per Cent*	*Number*	*Per Cent*
All A. S. A. members reporting affiliation with other societies:	1854	100.0	787	42.5	1082	58.4
Economic associations:	965	52.0	392	21.1	505	27.2
American Economic Association (1)	729	39.3	270	14.6		
American Marketing Association (4)	215	11.6	73	3.9		
Econometric Society (6)	173	9.3	13	0.7		
American Farm Economic Association (12)	76	4.1	22	1.2		
Other economic associations	102	5.5	14	0.8		
Political and social science associations:	643	34.7	158	8.5	222	12.0
American Academy of Political & Social Science (5)	196	10.6	22	1.2		
Academy of Political Science (7)	152	8.2	23	1.2		
American Public Health Association (8)	149	8.0	48	2.6		
Population Association of America (9)	134	7.2	12	0.6		
American Sociological Society (10)	102	5.5	15	0.8		
American Political Science Association	38	2.0	4	0.2		
Rural Sociological Society	23	1.2	1	0.1		
Other political and social science associations	187	10.1	33	1.9		
Mathematical and statistical associations:	417	22.5	107	5.8	182	9.8
Institute of Mathematical Statistics (2)	313	16.9	70	3.8		
Actuarial Society of America	35	1.9	1	0.1		
Institute of Actuaries	34	1.8	2	0.1		
National Society of Quality Control Engineers	33	1.8	7	0.4		
Other mathematical and statistical associations	203	10.9	27	1.5		
Business management associations:	210	11.3	49	2.6	56	3.0
Society for the Advancement of Management (14)	50	2.7	10	0.5		
American Management Association	43	2.3	6	0.3		
National Association of Cost Accountants	40	2.2	15	0.8		
American Society for Public Administration	34	1.8	3	0.2		
Other business management associations	54	2.9	15	0.8		
Psychological and educational associations:	112	6.0	21	1.1	29	1.6
American Psychological Association (11)	77	4.2	14	0.8		
Psychometric Society (13)	57	3.1	4	0.2		
American Educational Research Association	36	1.9	1	0.1		
Other psychological and educational ass'ns.	13	0.7	2	0.1		
Other associations	421	22.7	60	3.2	88	4.8
American Ass'n. for the Advancement of Science (3)	272	14.7	16	0.9		
Physical and Biological Associations	241	13.0	44	2.4		

* 772 (41.6%) reporting members belonged to associations classified in two or more groups; 555 (29.9%) members were associated with two groups; 180 (9.7%) with three groups; and 37 (2.0%) with four or five groups.

of those members who were affiliated with other societies belonged to associations which fall into two or more subject matter categories, and 12 per cent were members of societies classified in three or more of the general groups. Three hundred members (16 per cent) held col-

lateral memberships in an economic, political, or other social science association; almost 9 per cent were associated with both a political and social science association and an organization classified under "other associations"; and 7 per cent were members of both "economics" and "business management" groups.

Table 8 shows collateral memberships held by American Statistical Association members in any two of fourteen other professional societies which were checked most often by reporting members. For example,

TABLE 8

OVERLAPPING MEMBERSHIPS OF AMERICAN STATISTICAL ASSOCIATION MEMBERS' AFFILIATION WITH ONE OR MORE OF 14 OTHER PROFESSIONAL SOCIETIES IN 1945

	Professional Societies	Number and Percent of A.S.A. Members Affiliated with Both Societies														
		15	14	13	12	11	10	9	8	7	6	5	4	3	2	1
		A.S.A.	S.A.M.	P.S.	A.F.E.A.	A.P.A.	A.S.S.	P.A.A.	A.P.H.A.	A.P.S.	E.S.	A.A.P.S.S.	A.M.A.	A.A.A.S.	I.M.S.	A.E.A.
1	A.E.A.	729	19	3	40	3	19	30	12	103	106	101	96	62	44	
2	I.M.S.	313	3	29	2	13	9	11	14	6	56	13	10	80		2.4
3	A.A.A.S.	272	1	28	6	33	32	41	44	24	39	39	16		4.3	3.3
4	A.M.A.	215	16	4	7	5	5	3	3	22	16	23		0.9	0.5	5.2
5	A.A.P.S.S.	196	4	0	6	1	23	29	22	46	17		1.2	2.1	0.7	5.4
6	E.S.	173	2	12	10	6	5	12	4	21		0.9	0.9	2.1	3.0	5.7
7	A.P.S.	152	3	1	7	2	4	7	6		1.1	2.5	1.2	1.3	0.3	5.6
8	A.P.H.A.	149	0	1	0	4	13	33		0.3	0.2	1.2	0.2	2.4	0.8	0.6
9	P.A.A.	134	2	4	3	3	56		1.8	0.4	0.6	1.6	0.2	2.2	0.6	1.6
10	A.S.S.	102	3	4	3	5		3.0	0.7	0.2	0.3	1.2	0.3	1.7	0.5	1.0
11	A.P.A.	77	1	25	0		0.3	0.2	0.2	0.1	0.3	0.1	0.3	1.8	0.7	0.2
12	A.F.E.A.	76	0	0		—	0.2	0.2	—	0.4	0.5	0.3	0.4	0.3	0.1	2.2
13	P.S.	57	1		—	1.3	0.2	0.2	0.1	0.1	0.6	—	0.2	1.5	1.6	0.2
14	S.A.M.	50		0.1	—	0.1	0.2	0.1	—	0.2	0.1	0.2	0.9	0.1	0.2	1.0
15	A.S.A.	1854	2.7	3.1	4.1	4.2	5.5	7.2	8.0	8.2	9.3	10.6	11.6	14.7	16.9	39.3

NOTE: Percentages are shown in the lower half of this table and can most easily be related to the number of members by use of row and column numbers. The complete name of each professional society can be determined by comparing its code number with the code numbers given in Table 7.

of the 729 members who belonged to the American Economic Association, 106 also belonged to the Econometric Society and 103 were members of the Academy of Political Science. Duplications exist in these figures.[6] Among those societies not shown on this table, the most important overlapping occurred between the two actuarial societies listed, 25 American Statistical Association members belonging to both organizations.

[6] It is not possible to present in this paper the many combinations of societies to which American Statistical Association members belonged. However, data for any particular combination are available and can be had by writing to the National Office.

EDUCATION

More than 92 per cent of the American Statistical Association members have received degrees from colleges or universities, the highest degrees attained being divided almost equally between bachelor of arts, master of arts, and doctor of philosophy or their equivalents.

Fifty-seven per cent of the membership had degrees in one of the social sciences or their specialized disciplines; 65 per cent of those who held doctorates had a major in the social sciences. As would be suggested from previous tables, most of the members not only have B.A. degrees in economics, but one in eight have doctorates in this field.

One-fourth of the members have educational majors of mathematics and statistics; one in seven has doctorates in these two fields. Table 9 shows the major field of study for those members who reported that they had received one or more degrees.

TABLE 9

EDUCATIONAL MAJORS AND DEGREES OF AMERICAN STATISTICAL ASSOCIATION MEMBERS IN 1945

Education Major	Members Who Reported That They Received Degrees		Highest Degree Received or Equivalent					
			B.A.		M.A.		Ph.D.	
	Number	Per Cent	Number	Per Cent	Number	Per Cent	Number	Per Cent
All A.S.A. members reporting degrees*	2563	100.0	861	33.6	910	35.5	792	30.9
Economics	915	35.7	266	10.4	307	12.0	342	13.3
Mathematics	339	13.2	160	6.2	100	3.9	79	3.1
Statistics	269	10.5	92	3.6	122	4.8	55	2.1
Business administration and marketing	161	6.3	55	2.1	96	3.7	10	.4
Sociology	110	4.3	19	.7	39	1.5	52	2.0
Engineering	106	4.1	78	3.0	23	.9	5	.2
Psychology	90	3.5	11	.4	22	.8	57	2.2
Social science	69	2.7	23	.9	27	1.1	19	.7
Liberal arts	57	2.2	44	1.7	12	.5	1	†
Education	54	2.1	10	.4	24	.9	20	.8
Chemistry	54	2.1	19	.7	10	.4	25	1.0
Biology	46	1.8	14	.5	7	.3	25	1.0
Medicine	42	1.6	1	†	—	—	41	1.6
Political science	41	1.6	9	.4	17	.7	15	.6
Law	37	1.4	—	—	37	1.4	—	—
Agriculture	31	1.2	8	.3	13	.5	10	.4
Public health	26	1.0	2	.1	11	.4	13	.5
Physics and geology	22	.9	7	.3	6	.2	9	.4
Other studies	94	3.7	43	1.7	37	1.4	14	.5

* 208 of the 2724 members who reported education did not receive college or university degrees.
† Less than .05 per cent.

PARTICIPATION OF THE MEMBERS IN THE ACTIVITIES OF THE AMERICAN STATISTICAL ASSOCIATION

One out of every seven members has had papers or articles published in the JOURNAL OF THE AMERICAN STATISTICAL ASSOCIATION. However, more than half have published articles in other journals or books. Thirteen per cent of the membership have given papers at annual meetings and are in general the same members who wrote for the JOURNAL. Four per cent of the membership have held one or more national offices, and seven per cent have been officers in one of the local American Statistical Association chapters. Three per cent indicated that they had been chairman of one or several sessions of annual meetings.

GEOGRAPHICAL DISTRIBUTION OF THE MEMBERSHIP OF THE AMERICAN STATISTICAL ASSOCIATION

The change that has occurred in the geographical distribution of the Association's membership over the years is shown in Table 10. The per

TABLE 10

SHIFT IN GEOGRAPHICAL DISTRIBUTION OF AMERICAN STATISTICAL ASSOCIATION SINCE 1893

Area	1893	1926	1940	1945
Number of members	504	629	2644	3307
Per cent of members in				
New England	36.4	11.9	6.4	4.9
Middle Atlantic	28.4	45.8	35.6	31.5
South Atlantic	10.5	13.4	29.3	31.2
East North Central	12.7	18.6	13.3	12.4
Other areas	12.0	10.3	15.4	20.0

Data for 1893, 1926, and 1940 from table in article by Funkhouser, *op. cit.*, p. 339.
New England: Me., Vt., N. H., Mass., R. I., Conn.; Middle Atlantic: N. Y., Pa., N. J.; South Atlantic; W. Va., Va., Md., Del., N. C., S. C., Ga., Fla., D. C.; East North Central: Wis., Ill., Ind., Ohio, Mich.

cent of the membership in the New England region has declined steadily since 1893, while the proportion of the members in the South Atlantic region, which includes the metropolitan area of Washington, has continued to increase. In 1945 about 63 per cent of the members were located in the Middle and South Atlantic regions; 27 per cent lived in the Washington, D. C. area; and about one-fourth were located in New York state, chiefly in the City of New York. Women, comprising ten per cent of the total membership were distributed fairly proportionately among the areas.

The tables presented in this paper summarize the more important characteristics of the American Statistical Association membership. In the process of compiling these summaries all available information was coded and placed on punch cards, thus making further summarization or detailed analysis possible without undue additional effort. The file of punch cards will be maintained for six months and every effort will be made during that period to satisfy any considered request of individual members for additional information.

BRITAIN'S NEW POST-WAR ECONOMIC GUIDE

NORMAN CRUMP
Financial Editor "Sunday Times" (London)

NEARLY two years ago Britain's Government of the day, which included representatives of all political parties, issued a White Paper on Employment Policy. The concluding passages of this document made it clear that all the Government's proposals for the maintenance of employment depended for their success upon a full and continuous knowledge and appreciation of all facts bearing upon the nation's economic position. It, therefore, went on to say that "the Government intend to establish on a permanent basis a small central staff qualified to measure and analyze economic trends and submit appreciations of them to the Ministers concerned."

Actually such a staff came into existence early in World War II. It is described as the Central Statistical Office, and was directly responsible to the War Cabinet. Much of its work necessarily remained secret during the war, but it was responsible for the annual White Paper measuring and analyzing Britain's national income, as well as for the document describing Britain's war effort, which appeared towards the end of 1944.

Quite rightly, it is to remain a permanent body. One of the first fruits of its post-war labors is a new publication called the *Monthly Digest of Statistics*. This is available to the public, as well as to Ministers and Government offices. The first issue, for January, 1946, appeared in the middle of February.

Part of the information which it contains was already appearing elsewhere. For years past the Bank of England had published a monthly *Statistical Summary*, which was of the utmost use to those concerned in economic trends. Publication of this *Summary* has now ceased, and many of the statistics which it included have been transferred to and will be continued by the new publication. Similarly the *Board of Trade Journal* publishes statistics of wholesale prices, external trade, coal and steel production, etc. These again are to be found in the new document. So, too, with the statistics of wages, earnings, employment and the cost of living which appear each month in the Ministry of Labor's Gazette.

Here the main advantage is that these statistics are assembled in a single book. How great an advantage that will prove will be realized by all those who have to look up and interpret a particular statistical fact, while working against the clock. But the Digest does much more than assemble material which either is or was available elsewhere. It

includes many new tables, which were originally compiled for Government use during the war, but which for security reasons have hitherto remained unpublished.

The *Digest* falls into eleven main sections. These are Employment; Fuel and Power; Raw Materials (production, consumption and stocks); Manufactured Goods; Building; Agriculture and Food; External Trade; Merchant Shipping; Inland Transport; Finance (Government and Banking); and Wages and Prices. The tables are not yet entirely complete. In some instances stocks are not included. In others, pre-war statistics are either lacking or have been omitted. It is to be hoped that this will be remedied in subsequent issues, but in some cases it must be recognized that the necessary details only began to be collected during the war.

These are small points of criticism. It is much more important to notice that the raw materials section includes iron and steel in considerable detail; non-ferrous metals; timber; textiles (production, consumption and stocks); hides, skins and leather; rubber, both natural and synthetic; chemicals; and wood-pulp and paper. Manufactured goods statistics range from motor cars to sewing machines and electric irons; from fountain pens to tooth brushes; from dustbins to footwear; and wardrobes to dining-room chairs. There are details of beer, spirits and tobacco, wireless licences and car registrations; retail sales of food, clothing and household goods, in index form. The Building section includes the number of operatives at work, production of materials and components, temporary houses, and the progress (still in the initial stages) made towards the building of permanent houses.

Of greatest importance perhaps is the section on Employment. This is good as far as it goes. But the latest statistics are those for November, 1945, and demobilization has since made further considerable progress. Nor does the *Digest* contain any attempt to look ahead. There is nothing resembling the "man-power budget," advocated in the Employment Policy White Paper as an instrument essential to the proper formulation of employment policy.

That is not the fault of the Central Statistical Office. The preparation of a man-power budget depends upon decisions of the highest policy. But until those decisions are taken, and the man-power budget is prepared and published, the new *Statistical Digest* can only be an auxiliary, even though invaluable within its own limits. It will not by itself enable either Government Departments or industrialists to complete their future plans.

PROBLEMS AND METHODS OF THE SAMPLE SURVEY OF BUSINESS*

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In business sampling, as in the general sampling problem, the over-all test that we apply to a sample design is that it shall yield the desired information with the reliability required at a minimum cost; or, conversely, that at a given cost it shall yield the estimates desired with the maximum reliability possible. A second criterion which we impose on the sample design is that the reliability of the sample results should be measurable. This requirement eliminates from consideration many superficially attractive sampling schemes which, while generally economical in both time and money, may lead to faulty conclusions.

A particularly important characteristic of the distribution of sales of business establishments is that the distribution is highly skewed, with relatively few large establishments accounting for a substantial proportion of total sales. The sample design takes advantage of this fact and in so doing yields a sample of greater efficiency than would result from a sample of equal size if the distribution were normal. Various size classes are distinguished and the theory of optimum allocation indicates the number of stores to be sampled from each class. Large stores will be sampled from a listing of establishments, and the more numerous smaller stores will be sampled by cluster sampling.

The appropriate sampling theory is given for measuring the various contributions to the sampling error, and for maximizing the information obtained per dollar expended.

THE APPLICATION of recent advances in sampling methods to the problem of measuring, for example, retail sales and changes in retail sales by kinds of business now makes it possible to improve the quality of the current survey work being done in the business field by the Bureau of the Census. The revision of the survey methods will be carried out in connection with the expansion of this work now planned. The revision and expansion of the survey involves designing a sample of retail stores that will produce: (a) reliable national estimates of total retail sales and of month to month changes in sales, by kind of business; and (b) similar figures with somewhat less precision for individual large cities, and subsequently, for regions and individual States. In this

* This paper was presented at the annual meeting of the American Statistical Association on January 26, 1946 in Cleveland, Ohio. The authors wish to thank Mr. Max A. Bershad for reviewing the manuscript and making numerous helpful suggestions.

paper we shall consider primarily the design of the sample to meet the first of these objectives. The extension of the sample to provide statistics for individual cities and for other areas will be made in such a way as to further improve the general sample, but these methods will not be discussed here.

SOME FUNDAMENTAL CONSIDERATIONS IN SAMPLE DESIGN

In business sampling, as in the general sampling problem, the overall test that we apply to a sample design is that it shall yield the desired information with the reliability required at minimum cost; or conversely, that at a given cost it shall yield the estimates desired with the maximum reliability possible. Various resources may be available, and restrictions and limitations other than cost restrictions may necessarily be imposed on the design. The resources may be in the form of available statistical data, lists of establishments, an existing field organization, tabulation facilities, existing knowledge of the problem being dealt with, etc., together with available mathematical statistical tools. The restrictions and limitations may have to do with time limits to be met, limitations on kinds of personnel that can be used, etc. Designing the sample to achieve the maximum information at minimum cost involves, of course, making fullest possible use of resources and operating within the necessary restrictions.

A second criterion that we impose on sample designs in the Bureau is that the reliability of the sample results should be measurable. Methods of sample selection and estimation are available for which the risk of errors in the sample estimates can be measured and controlled. As the size of the sample is increased, the expected discrepancies between the estimated value from the sample and the true value (i.e., the value that would be obtained from a complete census) will decrease if such methods are used. With such methods one can know the risk taken that the error due to sampling will exceed a specified amount. This risk (i.e., the risk that the error will exceed any specified amount) can be made as small as desired by taking a sample of adequate size.

The use of methods yielding results whose sampling errors can be measured has these advantages: we can indicate to the users of our data the degree of confidence they can place in the published figures; we can make use of an objective criterion for choosing the "best" from among alternative designs; and we can determine in advance of drawing the sample whether resources are available to get results from the sample with the reliability required, and so we may be able to prevent useless sample surveys from being carried forward.

If we permit the use of sample designs that do not provide measurable results, a great many superficially attractive sampling schemes are eligible for selection. For obtaining a sample of a given size these methods are generally economical, in both time and money, but they may lead to faulty conclusions.

In referring to sampling errors or reliability of sample results here the reference, of course, is to the discrepancy between a sample result and the results that would have been obtained from a complete census carried out with equal care. Problems of accuracy of response are thus not being considered in this paper. It should be pointed out, however, that in practice it may be possible with a sample to carry out more careful operations and insist on greater accuracy in the various field steps than is feasible in a complete census, so that actually nonsampling errors may be reduced in a carefully conducted sample survey below what would be present in a complete census.

DIFFICULTIES OF METHODS OFTEN USED IN BUSINESS SAMPLING

A method often used in business sampling is to mail questionnaires to all or a sample of establishments included on available lists and to depend primarily on a voluntary mail response to the mailed inquiry. In using such returns it has usually been assumed that, if sales for two periods of time are collected from an identical set of stores, the change in sales between the two periods for the identical stores will, in fact, reflect the change in total sales for the specified kind of business. This type of sample estimate may be subject to two sources of bias: (1) the mail responses may not reflect the change in sales of the nonrespondents to the mail questionnaire; (2) the available mailing lists are usually out of date and hence the sample drawn from the lists does not provide the information for estimating the net effect of the turnover in business establishments.

It is possible to overcome the kind of difficulties outlined by the use of modern sampling methods. Using these methods we can produce results the reliability of which can be controlled, and which, with an adequate size of sample, can be made as precise as desired. At the same time, these methods point the way to minimizing the cost of obtaining results with the desired precision.

CHARACTERISTICS OF THE POPULATION TO BE SAMPLED AND RESOURCES AND FACILITIES AVAILABLE

A particularly important characteristic of the distribution of sales of business establishments is that the distribution is highly skewed. A

relatively few large establishments, constituting a small proportion of all establishments, account for a substantial proportion of the total sales. At the same time, the very large number of remaining establishments also account, in the aggregate, for a substantial proportion of total sales. This highly skewed distribution can be utilized in the sample design to produce more reliable results for a given size of sample than would be possible, if, say, the sales were normally distributed. The advantage of the skewness of the population can, of course, only be utilized if approximate measures of size are available in advance of taking the survey, thus making it possible to select the larger establishments for special treatment in sampling. This type of information is available for the larger establishments, although no up to date and complete list of all retail establishments is available.

Records that are available and will be used include: (a) Social Security records—the Social Security Board has records of employers in covered industries having one or more employees. These records are available on a relatively up to date basis and are classified by primary kind of business. (b) The last census of business—the 1940 Census provides a record of establishments by kinds of business as of the date of the census, although these records are not now up to date. However, the census provides extensive statistical information on retail trade as of 1940 that can be effectively utilized.

In addition to the Social Security records and the data from the 1940 Census, another important resource available to the Bureau of the Census is an existing field organization operating in 68 different sample areas. This field organization has been established for purposes of taking the Monthly Report on the Labor Force and can be used and extended for the current business survey.

An additional resource of importance is the availability in the Census of comparatively up to date maps covering cities having more than 25,000 population. These maps are brought up to date annually and show, for the most part, the location and general type of use of individual structures. They have been acquired by the Census as an aid in sampling and in census taking, and can be effectively utilized in sample surveys of both population and business.

We shall not attempt to present a detailed description of the sample, as many of the details will be modified as the design is put into effect, but a brief description of the sampling plan will be given with elaboration of one or two specific points as illustrative of the role that modern sampling methods play in determining the sample design.

DIVISION OF RETAIL TRADE INTO TWO SUBUNIVERSES AND ALLOCATION OF SAMPLING TO EACH

It will be convenient to regard the universe of all retail stores in the United States as divided into two subuniverses: first, those that are included on a list of large stores that will be prepared; and second, those that are not. One can proceed separately to design a sample for each of these subuniverses.

A question that must be considered concerning the subdivision of the population into a subuniverse of listed large stores and a subuniverse of all other stores is: How many establishments should be included on the large store list? Records are available from which more than half of all stores could be included on the list, whereas it is expected to include only about the largest 10 per cent. Why include only 10 per cent? Certain optimum sampling principles, which will be described later, dictate that the very large stores must be on the list and suggest that the more stores included on the list the better. On the other hand, various administrative considerations, such as the difficulty of identifying and matching stores in order to avoid including some in both subuniverses, together with problems of dealing with births and deaths of establishments, operate to hold the number of stores included on the list to a minimum. At the outset, these administrative factors point to keeping the list down to the 10 per cent now contemplated.

The list of large establishments, classified by kind of business, has been prepared based on the Social Security records. Each establishment on the list has an indication of size based on payrolls during a previous period. Of course, the actual current size of each establishment listed is not known, but the wages paid during a prior period provide an approximate measure of size.

The use of past payroll information to distinguish large from small stores means that there will be a number of stores not on the list that will be larger in current sales than many of those that are on the list. Despite this fact the use of a subuniverse of listed large stores will be highly effective in increasing the reliability of the sample. With the methods to be followed, if the measures of size used are highly correlated with the actual sales during the current periods under consideration (as will be the case), then the use of the measures of size in classifying the establishments will be exceedingly effective in reducing the sampling error. On the other hand, even if the measures of size were not highly correlated with actual sales, no bias would be introduced into the sample. Thus, the design is such that effective use has been

made of available resources so that their use increases the sample efficiency without in any way biasing the sample.

Considerations of optimum allocation[1] make it clear that the subuniverse of listed large establishments should be sampled more intensively than those not on the list. Consequently, approximately 30 per cent of those on the list will be included in the sample, while only about 2 per cent of those not on the list will be included. This allocation of sampling between the two subuniverses arises because the stores listed, while constituting only about 10 per cent of the total stores, account for about 60 per cent of the total sales. Conversely, those not on the list account for about 90 per cent of the stores and 40 per cent of the total sales. Actually, by including about 30 per cent from those on the list, and about 2 per cent from the remaining stores, we find that the number of establishments included in the sample from each of the classes will be roughly proportionate to the aggregate sales of each, rather than proportionate to the number of stores. This is in accord with optimum sampling principles where the principal characteristics to be estimated from the sample are total sales or changes in total sales.

To illustrate this principle, suppose we have stratified the population of establishments into two strata, and want to allocate the sampling to each stratum in such a way as to produce the most reliable results possible per dollar of cost. Suppose, also, that we want to estimate the ratio of sales in the current month to those in the prior month from the sample. We shall use as our estimate the ratio

$$(1) \qquad \frac{\frac{N_1}{n_1} X_1 + \frac{N_2}{n_2} X_2}{\frac{N_1}{n_1} Y_1 + \frac{N_2}{n_2} Y_2},$$

where X_1 represents, for the first stratum, the aggregate sales during the current month of the stores in the sample, and X_2 is the corresponding figure for the second stratum. Similarly, Y_1 and Y_2 represent, for the two strata, respectively, the aggregate sales of the same stores during the prior month. The total number of stores in the first stratum is N_1, and n_1 is the number of these included in the sample, while N_2 and n_2 are the corresponding figures for the second stratum. The vari-

[1] The notion of optimum allocation of a sample was introduced by J. Neyman, see "On the Two Different Aspects of the Representative Method; A Method of Stratified Sampling and the Method of Purposive Selection," *Jour. Royal Stat. Soc.*, New Series, Vol. 97 (1934), pp. 558–606. See also P. C. Mahalanobis, "A Sample Survey of the Acreage under Jute in Bengal," *Sankhyā*, Vol. 4 (1940), pp. 511–530.

ance of this sample estimate is given approximately by:

$$(2) \qquad \sigma^2_{\text{ratio}} = \frac{N_1 - n_1}{(N_1 - 1)n_1} \frac{Y_1^2}{Y^2} S_1^2 + \frac{N_2 - n_2}{(N_2 - 1)n_2} \frac{Y_2^2}{Y^2} S_2^2.$$

The first term above represents the contribution of the first stratum to the over-all variance. In this term, Y_1 is the aggregate sales of all establishments in the stratum during the prior month and S_1^2 is roughly equivalent to the variance between individual establishments in that stratum of the "sales ratio." By the "sales ratio" is meant the ratio, for an establishment, of the sales in the second period to those in the first. The second term has a similar meaning for the second stratum. Finally, $Y = Y_1 + Y_2$; that is, Y is equal to the aggregate sales during the prior period of all establishments in both strata combined.[2]

We want to obtain a sample that produces results of specified reliability at minimum cost, and to do this we must take account, also, of the relative cost of including an establishment in the sample from the first stratum as compared with the cost of including one from the second. Suppose, therefore, that the cost of including an establishment from the first stratum is k times the cost of including one from the second (k may be either greater or less than 1).

With these facts we can now ascertain the relative size of sample that should be taken from each stratum so as to produce results of the maximum accuracy per dollar of cost. We accomplish this by setting

$$(3) \qquad \frac{n_1}{n_1 + n_2} = \frac{Y_1 S_1}{Y_1 S_1 + Y_2 S_2 \sqrt{k}}.$$

It is apparent from this that if k is approximately equal to 1 (that is, if the cost of including an establishment in the sample is about the same

[2] Actually, $S_1^2 = \sigma_1^2 + (R_1 - R)^2 V_{1Y}^2 + B_1$, where $\sigma_1^2 = (V_{1X}^2 + V_{1Y}^2 - 2\rho_1 V_{1X} V_{1Y}) R_1^2$ is approximately the variance of the sales ratio for establishments in the first stratum; $R_1 = \frac{X_1}{Y_1}$, and $R = \frac{X_1 + X_2}{Y_1 + Y_2}$, with X_1 equal to the aggregate sales of all establishments in the first stratum during the current month, and X_2 similarly defined for the second stratum;

ρ_1 is the correlation between sales in the two periods for establishments included in the first stratum;

V_{1X}^2 is the coefficient of variation of the sales during the current period of the establishments in the first stratum, and

V_{1Y}^2 is the corresponding coefficient of variation for the prior period;

$B_1 = 2(R_1 - R)R_1(\rho_1 V_{1X} V_{1Y} - V_{1Y}^2)$, and B_1 will usually be of negligible size relative to the other terms of S_1^2.

S_2^2 is similarly defined but relates to establishments in the second stratum.

If more than two strata were used the variance of the sample estimate would be for, say, L strata

$$\sigma^2_{\text{ratio}} = \sum_{i=1}^{L} \frac{N_i - n_i}{(N_i - 1)n_i} \frac{Y_i^2 S_i^2}{Y^2},$$

where the terms for the ith stratum would all be defined as above for the first stratum.

for both of the strata), and if $S_1^2 = S_2^2$, which is approximately true when the standard deviation of the sales ratio is the same for establishments in the two strata, the optimum proportion of stores to be included in the sample from each stratum is equal to the proportion of the aggregate sales accounted for by the establishments in the respective strata.

It is such considerations that determined the relative allocation of sampling indicated earlier between the subuniverse of listed large establishments and the remaining stores. Moreover, it will be found that by using this optimum allocation method, the sampling error will be reduced very substantially below what would result from proportionate sampling from each subuniverse. Of course, not all of the facts necessary to determine an optimum job are available in advance, and adjustments will be made in the sample with further experience. However, knowledge of the principles and of the general characteristics of the population to be sampled makes it possible to approximate the optimum in the initial design.[3]

The theory just presented not only indicates the optimum sampling ratio to be applied to each of two size strata, but suggests that still more gains might be achieved by additional stratification by size of establishment and by using increasingly higher sampling ratios as the size of store increases. Such considerations suggest, also, that all stores larger than some particular size are of sufficient importance that they should be included in the sample. This theory, therefore, has a bearing on the question discussed earlier concerning how many stores should be included in the subuniverse of listed large stores. The relevance of the theory just presented to this latter issue arises because, while it is feasible to use sampling ratios ranging from 100 per cent down in sampling from various classes of stores included on the large store list, the method of sampling from the remaining stores is such that only small ratios are practicable. In fact, it is anticipated that at the outset not more than a 2 per cent sampling ratio will be applied to any sizes or classes of stores not included on the large store list.

THE SAMPLE FROM THE SUBUNIVERSE OF LISTED LARGE STORES

In the previous section we described how retail stores were divided into two subuniverses, a list of large stores, and all stores not on the

[3] It should be recognized that the theory of optimum sampling just outlined holds for any single statistic, but where estimates of a number of different items are wanted from a sample a compromise may have to be made between the designs which would be optimum for each of the various statistics involved. Often, however, as is the case here, many of the items for which estimates are wanted will be highly correlated, and for these the design is close to the optimum for each.

list. We shall now consider the sample from the list of large stores. The variation among the sizes of even those stores on this list is very great and consequently one should not take a uniform sampling ratio from these if he is to approximate an optimum sample design.

The principles discussed in the previous section suggest that optimum allocation would call for including in the sample all stores having sales larger than a certain size, and that the large store list should be divided into a number of strata based on size of store with a different sampling ratio assigned to each. There are, however, serious disadvantages in having too many different sampling ratios. The more sampling ratios imposed, the larger the number of different weights that must be used. Moreover, most of the gains of optimum allocation to size groups will be achieved if we set up only two classes of large stores, one consisting of the stores which will be included with certainty, and the other consisting of the remaining listed large stores. Once the list of large stores is prepared, therefore, it will be further divided into a *very large store list* from which we make every effort to obtain returns 100 per cent, and a *medium large store list* from which perhaps 20 per cent returns may be required.

Joint use of mail and personal follow up. Since each of the very large stores is of sufficient size that it should be included in the sample, no matter where located, this small group will be mailed to and followed up with field interviews as necessary to insure substantially complete coverage each month. The follow-up work will be facilitated by the fact that most of the very large establishments are concentrated in the larger cities and that a prompt mail response can be expected from most stores in this group.

A further application of the principle of optimum use of methods is made in the joint use of mail canvass and field follow-up methods in sampling from the medium large store list. The collection of a questionnaire by mail can be done at a fraction of the cost of collection by field interview—provided a mail response to the questionnaire can be obtained. Consequently, the fullest possible use of this method of canvass should be utilized with minimum use made of the much more expensive method of personal follow up.[4] The schedules to be mailed each month will be in excess of the number actually expected to be returned, and enumerative follow-up will be made of a sample of those that do not respond to the mail canvass. The plan is for a rotation

[4] See "The Problem of Non Response in Sample Surveys" (this JOURNAL) for a description of optimum methods of joint use of mail and enumerative response, or of any two methods where one is less expensive but the other is more certain of obtaining response.

in the follow-up of the medium large stores not responding so that over a period of a relatively few months all of those not responding to the mail canvass will be visited in an effort to build up cooperation to the mail survey. With the mail response and coverage through follow-up of a sample of those not responding, it is possible to compute sample estimates without assuming that those responding are representative of all medium large establishments.

Use of cluster sampling to facilitate field work by restricting sample to selected primary areas. Field follow-up of nonrespondents to a mail survey would be exceedingly expensive if the establishments to be covered were widely scattered throughout the entire United States. If it were not for this expense the optimum procedure would call for designating the establishments to be included in the sample with no clustering of the sample establishments whatever. To deal with the problem of cost, however, the device of clustering the sample will be used. A sample of primary areas will be designated and mailing to the medium-large stores will be limited to those located within the selected primary areas. A cross-section of such areas will be designated throughout the United States such that, if the retail stores within these areas are adequately represented in the sample, sufficiently reliable information for all stores will be available. This does not mean that the sample should be adequate to represent well each primary area separately. It does mean that the sample should be adequate to represent all primary areas combined.[5] Since the list of medium-large stores is not extensive, all the establishments on this list within these areas will be mailed to, and the follow-up of a sample of those not responding can be instituted. Those to be contacted will be rotated so that over a period of perhaps three months all nonrespondents to the mailed questionnaires will be visited.

Optimum choice of primary areas. With the adoption of this cluster sampling approach the question still remains: What kind of a sample of areas ought we to select? The solution to this problem was fairly clear in this instance because, as indicated earlier, the Bureau has an operating field organization taking the Monthly Report on the Labor Force. A somewhat similar problem presented itself in the design of the labor force sample. For that survey, a sample of 68 primary areas has been designated within which subsampling operations are carried on, and a full-time supervisor with part-time enumerators is available in each of these areas. Each primary area consists of a county, or, in some

[5] This important principle is often misunderstood. See, for example, the paper entitled "Design, Size, and Validation of Sample for Market Research," *Journal of Marketing*, January 1946, by the Committee on Marketing Research Techniques.

instances where the sizes of the counties are not too large, of two or three adjacent counties. The principles of sampling and the procedures followed in designing the labor force sample, described elsewhere,[6] are equally applicable to business sampling.

An obvious question to be answered before the adoption of these sample areas for business sampling is: What reason is there to presume that a sample of areas designated for such a population sample will prove to be a satisfactory set of areas in which to take a retail trade sample. The answer is not too difficult. In the first place, the design of the population sample was an unbiased design, from which it follows that unbiased estimates of both population and business as well as of other characteristics can be obtained by a survey of these areas. The real question then is: Will this sample produce results having a sufficiently small sampling error? Since retail trade exists to serve the population, it is not surprising to find that it is distributed between counties much as the population is distributed and, therefore, that a sample designed to estimate the labor force characteristics will be comparatively efficient also for estimating retail trade, if appropriate methods of estimation are used. It can be seen from the following table that the average sampling errors resulting from the use of a sample designed exactly as the labor force sample but used for retail trade will produce national estimates for many kinds of business with fairly high precision.

NATIONAL ESTIMATES OF RETAIL SALES FOR ALL RETAIL TRADE AND FOR A FEW KINDS OF BUSINESS FROM THE 68 AREA LABOR FORCE SAMPLE*

	Sales			
	Known 1939 Census	Estimate from sample	Per cent difference	Coefficient of variation of sample estimate in per cent
Total—all types	42,041,790	42,170,007	+0.3	1.4
General Stores (with food)	810,342	791,095	−2.4	12.6
General Mdse. Group	5,665,007	5,678,947	+0.2	2.5
Drugs	1,562,502	1,533,235	−1.9	2.0
Apparel	3,258,772	3,187,193	−2.2	2.0
Lumber-Hardware	2,734,914	2,563,790	−6.3	3.9
Furniture	1,733,257	1,657,949	−4.3	3.3

* Computations for this table were made for a special survey which covered only these six kinds of business. General Stores (with food) occur in the rural areas, but seldom in larger cities, and consequently, for the 68 area sample the sampling error for this special class is large.

[6] Morris H. Hansen and William N. Hurwitz, "A New Sample of the Population," *Bureau of the Census*, September, 1944.

This table gives an estimate of the sampling errors to be expected in estimating total sales for a few kinds of business from the 68 labor force sample areas—but on the assumption of complete coverage of stores within the designated areas. Thus, the table provides a guide to how representative these areas are of the entire nation with respect to retail trade. The per cent difference column shows the actual errors in estimating total sales from the sample counties, while the coefficient of variation indicates the average error to be expected over all possible samples drawn by the method actually followed.

We see that the labor force and general population sample may be expected to produce retail trade estimates of reasonable reliability, but it should be clear that at least some gains in precision, whether small or large, could be achieved if a sample of areas were utilized that was designed explicitly for retail trade rather than for some other problem. The question arises, then, why not design a new sample? The answer to this question has two aspects. The first is that, for the same size sample, some gains undoubtedly could be achieved by a more specialized design. Nevertheless, we have had considerable experience indicating that, on the average for the many different statistics to be estimated, the gains which would result would not be large. The second and principal point is that, with field facilities already existing in these counties, the cost of obtaining returns is substantially less by utilizing the available counties than by drawing an independent sample in other areas. Therefore, *per dollar*, the most reliable results are obtained in the way specified. If more reliable results are wanted, we need merely to expand that sample. Such considerations led to the adoption of the 68 area labor force sample as the areas in which mailing and field operations would be carried on; this sample subsequently will be enlarged by expanding to additional areas.

Principles involved in the determination of the number of primary areas to be covered and the size of sample in each. The proposed expansion to more areas suggests the need of some guide as to how many primary sampling areas should be listed in the sample and how large a sample should be selected within each. We shall indicate some principles that serve as a guide in making this determination, but shall present them in a very simplified problem. In particular, we shall consider only the problem of sampling from a single stratum, and we shall assume that the number of stores is the same for all counties in the stratum. The principles pointed up and conclusions arrived at for this simple situation are equally applicable to the more complex problem with which we are actually dealing.

Suppose that there are a total of M counties in a stratum, from which we take a sample of m at random; that there are a total of N stores in each county in the stratum, and that we subsample n from each selected county. Thus, we will have a sample of m counties from this stratum, and of n stores from each county, or a total sample of mn stores. If we compute the average sales per establishment for the stores in the sample the variance of this sample average will be:

$$\sigma^2_{\text{average}} = \frac{M-m}{(M-1)m}\left\{\sigma_b^2 - \frac{M-1}{M}\,\frac{\sigma_w^2}{N-1}\right\} + \frac{MN-mn}{M(N-1)}\,\frac{\sigma_w^2}{mn}$$

where σ_b^2 is the variance of the average sales per establishment between counties in the stratum, and σ_w^2 is the average within county variance.

The term $\left\{\sigma_b^2 - \frac{(M-1)\sigma_w^2}{M(N-1)}\right\}$ will, with very few exceptions, be positive in practical sampling problems. For a fixed total size of sample, that is, for fixed mn, the second term of the expression for the variance of the sample average will be unaffected by the way in which the sampling is divided between m and n. Therefore, for a sample of the total size of mn, the variance of the sample average will be smallest when the maximum possible number of counties is included in the sample and the sampling within counties is correspondingly reduced. Thus, if a sample of 500 stores is to be taken, i.e., if $mn=500$, the most reliable result will be achieved by taking only a single store from each of 500 sample counties. It follows that if there are not enough counties in the stratum to accomplish this, then the optimum sampling would call for including *every* county in the sample.[8]

With this fact established, why then do we propose to take a relatively small number of counties into the sample with a considerable number of stores from each? The reason is that we are trying to get the most reliable result possible per dollar spent, rather than per store included in the sample, and considerable economies can be effected by clustering the sample.

To illustrate this, let us go one step farther and assume that the total cost, C, of taking a sample can be broken down as follows:

$$C = c_1 m + c_2 mn.$$

This amounts to saying that there is an overhead of C_1 dollars that is incurred for each county that is included in the sample, and that a cost of C_2 is incurred for each store covered within a sample county.

This, again, is a much oversimplified assumption, but it will be useful in showing how the allocation of sampling as between including more counties and taking a larger sample within fewer counties may be affected by cost considerations.

Now, if we find the size of sample to be taken in each county that will produce the most reliable results possible per dollar spent we get

$$(5) \qquad n = \frac{C_1}{C_2} \frac{\sigma_w}{\sqrt{\left(\sigma_b^2 - \frac{\sigma_w^2}{N-1}\right)}}$$

as the sample to take within each county. Then m, the number of counties to be included, is the number that the budget allows or that is needed for the required accuracy. This theory provides a guide for determining how many counties to sample and how large a sample to include from each. As we learn more about the costs involved and the population we are sampling and its distribution between counties, this theory will be available to improve the sample design. At the outset, we know enough based on research already done and experience and general knowledge of retail trade to determine the first approximation that should be followed. That first approximation, in this case, is to start out initially with the sample of 68 primary areas used for the labor force sample, and to take a fairly large subsample within each primary area, although it is to be expected that the sample will be expanded to more areas subsequently.

THE SAMPLE FROM THE SUBUNIVERSE OF SMALLER STORES

We have up to this point a sample from only the subuniverse of listed large stores. For stores not on the list, which include the great bulk of establishments, we propose to confine our sample to the same primary areas that are being used for sampling the listed medium-large stores. In addition to the advantages of minimizing cost of travel already discussed, the use of these primary areas for the remaining stores makes it feasible to develop and maintain an up to date sample of births and deaths of stores as they occur.

Some of the primary areas include only a small number of stores, and in these areas it is feasible to prepare locally a complete list of establishments in the area and to be currently informed on births and deaths of establishments as they occur. For most of the primary areas, however, the stores involved are too numerous to make it feasible to list all stores locally and to follow all births and deaths of establish-

ments within the areas as they occur, and for these areas a method of subsampling from the selected primary areas will be used involving a further application of an area sampling procedure.

This area sampling procedure consists of designating small areas within the selected primary areas. The small areas will represent the stores not included on the large store list and the maintenance of up to date listings of stores in these areas will reflect the turnover in business establishments due to births and deaths.

The problem in designating small areas to be sampled within the selected primary areas is to locate and utilize the best available information and facilities for defining efficient subsampling units. The types of sampling principles already described as well as additional ones will be utilized in arriving at an efficient subsampling design.

For cities of 25,000 population or over the detailed Sanborn Insurance Maps referred to earlier provide an excellent mapping source that can be used in defining subsampling units. These maps are brought up to date at frequent intervals—about once a year, and show the locations of most stores within blocks in all such cities. Using these maps, it is possible to approximate the number of stores in each block and draw a sample of blocks stratified by size. An alternative possibility which will be followed sometimes is to designate small areas which have, according to the map, one, two, three, or as many stores as we desire, to be used as subsampling units. Such areas can be designated for inclusion in the sample by giving each block a chance of coming into the sample which is proportionate to its size in terms of number of stores, and then each sample block so selected can be segmented into areas of approximately equal numbers of stores. One of these areas is chosen at random from each sample block for inclusion in the sample.[7]

The methods used in sampling these small areas are such that any errors in the number of stores shown on the map will not bias the sample, although the poorer the counts made from the maps, the larger will be the sampling variance. Actually, the counts of stores from the maps have been found to be of such a quality that substantial reductions in sampling error have been made possible through their use.

In cities of 2,500 to 25,000 population, the Sanborn maps are not being utilized. The particular cities of this size to be included in the sample are obtained by a subsampling process within the selected counties and the 1940 Census returns are used in designating the ones

[7] For the theory involved in sampling with probability proportionate to size, see, Morris H. Hansen and William N. Hurwitz, "On the Theory of Sampling from Finite Populations," *Annals of Mathematical Statistics*, December, 1943.

to be included in the sample. Each city has a chance of selection proportionate to the number of stores in that city according to the 1940 Census. In most of the counties, however, there are only a few if any cities of this class, and all of them in the county may be included in the sample. For the cities of this size that are in the sample a straight sample of city blocks is being drawn for enumeration outside of the downtown area of such city, and a complete listing of store addresses will be made for the downtown areas and a subsample of stores or of small areas will be designated for coverage based on the information obtained in this listing.

To obtain a sample of the cities under 2,500 population and of other rural areas, minor civil divisions are used as sampling units. For these, 1940 Census information on the number of stores in each area is not available, and rough estimates of number of stores in each minor civil division are computed. These estimates are based on the 1940 population of the areas and on the proportion of that population living inside and outside of towns and villages. Then the actual areas to be included in the sample are selected with probability proportionate to this measure of size, or are stratified on the basis of this measure of size as was indicated above for the selection of blocks in the larger cities.

For the cities of all sizes, once a set of small areas is drawn into the sample all stores in those areas are listed. The enumerator is given a map showing the boundaries of the areas and is instructed to canvass the areas and list every store included therein. The advantage of the area approach is that it gives all stores not on the large store list a chance of being drawn into the sample, including both new establishments and ones inadvertently omitted from the large store list.

METHOD OF ESTIMATION FROM THE SAMPLE

A fundamental part of a survey design is the method of estimation from the sample. Equally fundamental is the method of evaluating the reliability of the estimates. The main aspects of the method of estimation and for evaluating contributions to the sampling error for samples of the type here outlined is presented in another paper that describes the methods and principles of estimation for the labor force sample.[8] The data from the 1940 Census of Retail Trade will be effectively utilized in preparing estimates from the sample, and will be utilized in such a way as to increase substantially the reliability of the

[8] See reference cited in footnote 7; and see also reference cited in footnote 6.

sample estimates through the use of ratio or regression types of estimates.

SUMMARY REMARKS

There will necessarily be variations introduced into the design as the plans are developed and put into operation, but the main principles followed will be those described above. Once the work in the field has progressed over a period of time further variations in the design will be introduced. For example, procedures may be developed through experience for handling an extensive list of large stores expeditiously in which event the large store list will be extended considerably. Again, the number of primary areas covered may be extended to make possible regional or State estimates as well as more detailed and accurate national estimates. At the same time, changes will be introduced as called for through further study of the costs involved, the sources of contributions to the sampling error, and the problems arising in carrying out the survey in the office and in the field.

ACTUARIAL ANALYSIS OF THE OPERATING LIFE OF B-29 AIRCRAFT ENGINES

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This study describes the actuarial methods used to determine the probable service life of the R-3350 engines used in B-29 aircraft. The use of actuarial methods to determine the expectation of life of engines under actual operating conditions is clearly superior to that of using averages based on the flying hours attained solely by engines removed for overhaul. This superiority is greatest when the engine "age" distribution or operating conditions are changing.

Though these methods and procedures were used in planning R-3350 engine requirements, production and shipping schedules, they are generally applicable in determining the number of replacements necessary to maintain a predetermined schedule of operation and can also be used in setting rates of depreciation.

This article presents the number of hours R-3350 engines ran before removal for overhaul, measures the improvement in successive models of engines, determines the difference in life between new and overhauled engines, and shows the effect of different operating uses upon engine life.

IT WAS necessary, very early in the B-29 aircraft program, to estimate the expectation of life of R-3350 engines prior to removal for overhaul, and to develop sensitive measures of actual operating life in order to adjust these estimated expectations of life (planning rates) to experience. The expected number of operating or flying hours prior to overhaul was a major element in determining the number of engines required each month to support the B-29 training program in the United States and to make possible the planned number of combat and other tactical missions against the enemy. The life of these engines was also a major element in determining the number of flyable engines that would be available month by month to support the B-29 program. Thus, decisions with respect to R-3350 engine requirements and availability vitally affected a whole series of inter-dependent variables, including (1) the ultimate size and the rate of expansion of the B-29 striking force, (2) the number and length of missions of B-29 aircraft, and (3) the required amount and rate of expansion of R-3350 engine production

* The procedures and data described in this article were developed by Lt. Col. Altman and Mr. Goor while they were attached to the Analysis and Plans Branch, A-4, Headquarters Army Air Forces. The authors wish to thank three other members of the Branch for their contributions: Mrs. Nina W. Levin and Miss Catherine Gordon for their assistance in the preparation and analysis of the original data; and Lt. Col. Harold Lamberton for the preparation of all charts involved in these studies.

and overhaul facilities. Accurate decisions with respect to the expansion of production and overhaul facilities were particularly important. On the one hand, the relatively long time required to obtain, fabricate and assemble the materials and components made engine production schedules relatively inflexible. On the other hand, expansion of facilities required large amounts of critical tools, materials and labor. For these reasons, the actual and planned operating life of R-3350 engines under combat and training conditions was a particularly important determinant of the entire B-29 aircraft program and therefore of the long-range air attack on Japan.

Engines for B-29 aircraft were critically short in 1944 and the first half of 1945, and the supply was extremely tight even at the end of the war with Japan. For many months air transportation was used to rush R-3350 engines to combat units overseas and then back to the United States for overhaul. This was no small feat since the great bulk and weight of these engines made it impossible to carry more than two R-3350 engines in the largest Army Air Forces transport.

All these factors created great pressure for accurate measurement of engine life in order to make possible the most effective supply and distribution planning and control, and led to the establishment, for the first time in the Army Air Forces, of the record-keeping and field reporting of the data required for the accurate determination of engine life by actuarial methods.

The fundamental purpose of this actuarial analysis of R-3350 engine life was to measure the average length of life of these engines, as well as their average future lifetime (expectation of life) after any specified period of flying time. The basic data used were the ratios of the number of hours flown by each engine removed for overhaul, to the number of hours flown by *all* engines, including those not yet removed for overhaul. Until actuarial methods were adopted, engine life prior to overhaul was measured solely by the average number of hours flown by engines removed for overhaul.

The latter method of necessity incorrectly showed low average engine life for many months after the introduction of a new type of engine for two reasons: (1) When *all* engines have flown a relatively small number of hours, the average number of hours flown by *removed* engines must also be relatively small. Thus, if no installed engine in a group of operating aircraft has accumulated more than one hundred hours of flying time, any engine removed for overhaul from this group would have a maximum of one hundred hours, and the average number of hours on *all* engines removed would be considerably less than one hun-

dred hours. As the surviving engines of this group accumulate more hours, both the maximum and the average flying hours on removed engines would increase. (2) The "age" distribution of installed engines would be affected by the introduction of large numbers of new aircraft into the group. These new aircraft would have engines that have flown only a small number of hours. While the introduction of new aircraft would not affect the rise of the *maximum* number of hours at removal of engines in this expanded group, it would retard the increase in the *average* number of hours at removal. In short, the method of measuring the number of hours flown by engines prior to overhaul solely on the basis of engines removed for overhaul always yields incorrect results under changing conditions of warfare because the number and "age" distribution of installed engines is constantly changing. This method can yield accurate results only after the "age" distribution of installed engines has been stabilized, and then only if operating conditions continue unchanged. Until the "age" distribution becomes standardized, which may take a year or more under usual rates of build-up of the striking force to peak strength, the average number of hours flown by removed engines prior to their removal would rise month by month. In the meantime, use of these averages as a planning rate for the future or as a measurement of performance in the past would be seriously in error and hopelessly inflate engine requirements.

Actuarial methods make it possible, on the other hand, to calculate the expectation of life of engines, with substantial accuracy, both while the "age" distribution is changing and after it has stabilized.

BASIC DATA

The data were collected in greater detail as the actuarial method of analysis gradually gained more and more support. By the spring of 1945 the following data were being collected by a training air force in the United States each month:

1. Number of installed engines:
   a. On hand, beginning of period
   b. Arrived during period
   c. Lost during period
   d. Departed during period
   e. On hand, end of period
2. Number of engines removed for overhaul:
   a. For accident, modification or other arbitrary reason
   b. For "normal" engine failure

Each major using organization in the United States and overseas

TABLE 1
NEW ENGINES IN TRAINING OPERATIONS IN THE UNITED STATES

TABLE 1

OPERATING LIFE OF R-3350-23A NEW ENGINES IN TRAINING OPERATIONS IN THE UNITED STATES
APRIL 30–JULY 31, 1945

Line No.	Flying Hours Since Manufacture or Last Overhaul	Number of Installed Engines				Engines Removed for Overhaul During Period		Engines Exposed to Contingency of Failure	Per Cent Removed for Overhaul (Removal Rate)	Smoothed Data		
		On Hand at Beginning of Period	Arriving During Period	Lost During Period	Departing During Period	On Hand at End of Period	For Accident, Combat Damage, Modification, or Other Arbitrary Reasons	For "Normal" Engine Failure			Per Cent Removed for Overhaul	Per Cent of Original Engines Surviving	Expectation of Life (Hours)
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
(1)	0											100.0	310
(2)	1– 20	153	241	2	27	94	3	7	384.0	1.8	.8	99.2	293
(3)	21– 40	56	258	1	39	113		5	592.0	.8	1.0	98.2	276
(4)	41– 60	21	87		28	57	1	8	678.5	1.2	1.2	97.0	259
(5)	61– 80	41	58	5	46	44		6	682.5	.9	1.4	95.6	243
(6)	81–100	65	26	1	20	27	8	14	695.5	2.0	1.6	94.1	226
(7)	101–120	69	19	4	20	30	2	17	715.0	2.4	1.9	92.3	210
(8)	121–140	49	24		19	30	6	17	723.0	2.4	2.3	90.2	195
(9)	141–160	27	24	1	12	31		25	718.5	3.5	2.7	87.8	180
(10)	161–180	30	21	1	18	31	7	36	684.0	5.3	3.2	85.0	166
(11)	181–200	42	15	1	24	22	7	40	636.5	6.3	3.9	81.7	152
(12)	201–220	37	6		14	45	11	53	584.5	9.1	4.6	77.9	139
(13)	221–240	16	7		21	40	12	33	493.0	6.7	5.4	73.7	126
(14)	241–260	21	17		24	29	8	27	423.5	6.4	6.5	68.9	115
(15)	261–280	24	6	2	11	13	13	25	380.5	6.6	7.6	63.7	103
(16)	281–300	16	7	1	9	21	11	39	341.5	11.4	9.1	57.9	92
(17)	301–320	24	2		11	12	13	31	288.0	10.8	10.9	51.6	82
(18)	321–340	16	6		7	12	10	36	248.5	14.5	12.9	44.9	73
(19)	341–360	8	4		4	7	8	36	205.5	17.5	15.2	38.1	65
(20)	361–380	5			1	13	7	30	158.0	19.0	18.0	31.2	57
(21)	381–400	7	1	1		7	1	28	120.5	23.2	21.3	24.6	49
(22)	401–420	10	1		3	6		20	95.0	21.1	25.3	18.4	42
(23)	421–440	2	4		4	7	2	16	77.5	20.6	30.0	12.9	36
(24)	441–460	1			1	7		21	57.0	36.8	36.0	8.3	30
(25)	461–480					4		9	27.5	32.7	42.0	4.8	25
(26)	481–500					1		3	13.0	23.1	50.0	2.4	21
(27)	501–520							3	7.5	40.0	69.0	1.0	16
(28)	521–540							2	4.0	50.0	70.0	.3	10
(29)	541–560							1	2.0	50.0	84.0	—	—
(30)	561–580							1	1.0	100.0	100.0	—	—
	Total	740	833	20	363	722	129	589					

Note on method of computation: The method of computation is described in general terms in the text. This is illustrated below for line 21 of the above table:

The data in Columns 1–8 are given.

The entry 120.5 (Col. 9, line 21) equals (a) the sum of lines 21–30 in Column 8, or 104; plus (b) the net total of Column 6 minus Column 2 for lines 22–30, or 14; plus (c) one-half the net total of Column 6 minus Column 2 for line 21, or −1.5; plus (d) the net total of Column 4 plus Column 5 plus Column 7 minus Column 3 for lines 22–30, or 3; plus (e) one-half the net total of Column 4 plus Column 5 plus Column 7 minus Column 3 for line 21, or 1.

The entry 23.2 (Col. 10, line 21) equals 28 (Col. 8, line 21) divided by 120.5 (Col. 9, line 21) and multiplied by 100.

The entry 21.3 (Col. 11, line 21) was read from the smooth curve fitted to the crude removal rates, as described in the text.

The entry 24.6 (Col. 12, line 21) equals (a) 100.0 minus 21.3 (Col. 11, line 21), or 78.7; multiplied (b) by 31.2 (Col. 12, line 20); divided (c) by 100.

The entry 49 (Col. 12, line 21) equals (a) the sum of Column 12 for lines 22–30, or 48.1; multiplied (b) by 20, to give 96.2; divided (c) by 24.6 (Col. 12, line 21), to give 39; plus (d) one-half of 20 hours.

was required to report these data in 20-hour flying time groups, with time measured as the number of hours flown since manufacture or last overhaul. The data were reported separately for new and overhauled engines and for each model of R-3350 engine. (The various engine models represented various stages in the development and improvement of the engine, beginning with the unmodified R-3350-23A carburetor type engine used in training and in early operations from India, followed by the modified R-3350-23A carburetor type and then later by the R-3350-57 fuel injection type.)

Collection of the data in this detail has made it possible to compare the relative performance of the various engines models, the performance of the same model in different theatres of action and under different operating conditions, the relative performance of new versus overhauled engines and the change in performance of any one model through time.

METHOD OF ANALYSIS

From these data were calculated, in terms of 20-hour flying time groups, the number of engines exposed to the contingency of failure, the crude and smoothed per cent removed for overhaul, the per cent of original engines surviving and the expectation of life. A sample calculation is shown in Table 1. The general method of computation may be described as follows with references to the columns in Table 1, assuming that frequency distributions by 20-hour flying time groups were reported for each of the following items:

(1) Number of engines removed for " normal" engine failure (col. 8).
(2) Number of engines removed for accident, combat damage, modification or other arbitrary reasons (col. 7).
(3) Number of installed engines on hand at the beginning of the period (col. 2).
(4) Number of installed engines on hand at the end of the period (col. 6).
(5) Number of installed engines on aircraft that arrived within reporting jurisdiction during period (col. 3).
(6) Number of installed engines on aircraft lost during period (col. 4).
(7) Number of installed engines on aircraft that departed from reporting jurisdiction during period (col. 5).

The exposure in any hour group X, is as follows:

$$[\text{col. } 9]_x = \sum_{t=0}^{\infty} [\text{col. } 8]_{x+t} + \sum_{t=1}^{\infty} [\text{col. } 6 - \text{col. } 2]_{x+t} + \tfrac{1}{2}[\text{col. } 6 - \text{col. } 2]_x$$

$$+ \sum_{t=1}^{\infty} [\text{col. } 4 + \text{col. } 5 + \text{col. } 7 - \text{col. } 3]_{x+t}$$

$$+ \tfrac{1}{2}[\text{col. } 4 + \text{col. } 5 + \text{col. } 7 - \text{col. } 3]_x$$

where the column numbers refer to the items in Table 1, Col. 9 being the "Engines Exposed to Contingency of Failure."[1]

The crude removal rates were then computed as a percentage by dividing the number of engines removed for normal engine failure in each hour group by the exposure in that group.

Analysis of a large number of these crude engine removal rate curves indicated that they had similar trends. When these crude rates were plotted on semi-logarithmic paper, they approximated straight lines. As a simple and quick method of graduation was desired, the data were plotted on semi-logarithmic paper with hours on the arithmetic scale. A straight line was then fitted by inspection. Chart 1 illustrates the results obtained by this procedure.

The percentage of the original number of engines surviving to the end of an hour group was computed by subtracting the per cent removed in that hour group from 100.0 per cent and by applying the remainder divided by 100 to the percentage of engines that survived to the beginning of that hour group.

The average length of life or the average future lifetime was then obtained by using the standard procedure for calculating the "complete

[1] This formula is an adaptation of the general "exposed to risk" formulae used in the calculation of mortality tables for human lives. The following are the basic elements in the construction of the formula:

(1) Every engine that reached an hour group greater than X during the period of observation must have passed through the hour group X during the period of observation unless it began the period or entered the experience during the period, at an hour group greater than X.

(2) Therefore, for any hour group X, the formula counts the number of engines that passed through that hour group during the period of observation as the sum of (a) the engines removed for "normal" engine failure at hours greater than X, during the period of observation; plus (b) the engines deducted from the experience because of "other" reasons for removal, loss while installed in aircraft or departing while installed in aircraft, at hours greater than X, during the period of observation; plus (c) the engines still installed at the end of the period of observation with hours greater than X.

(3) From the total in (2), above, are subtracted the engines that could not possibly have passed through the hour group X during the period of observation because they (a) started the period of observation as installed engines at hours greater than X; and (b) arrived installed in aircraft during the period of observation at hours greater than X.

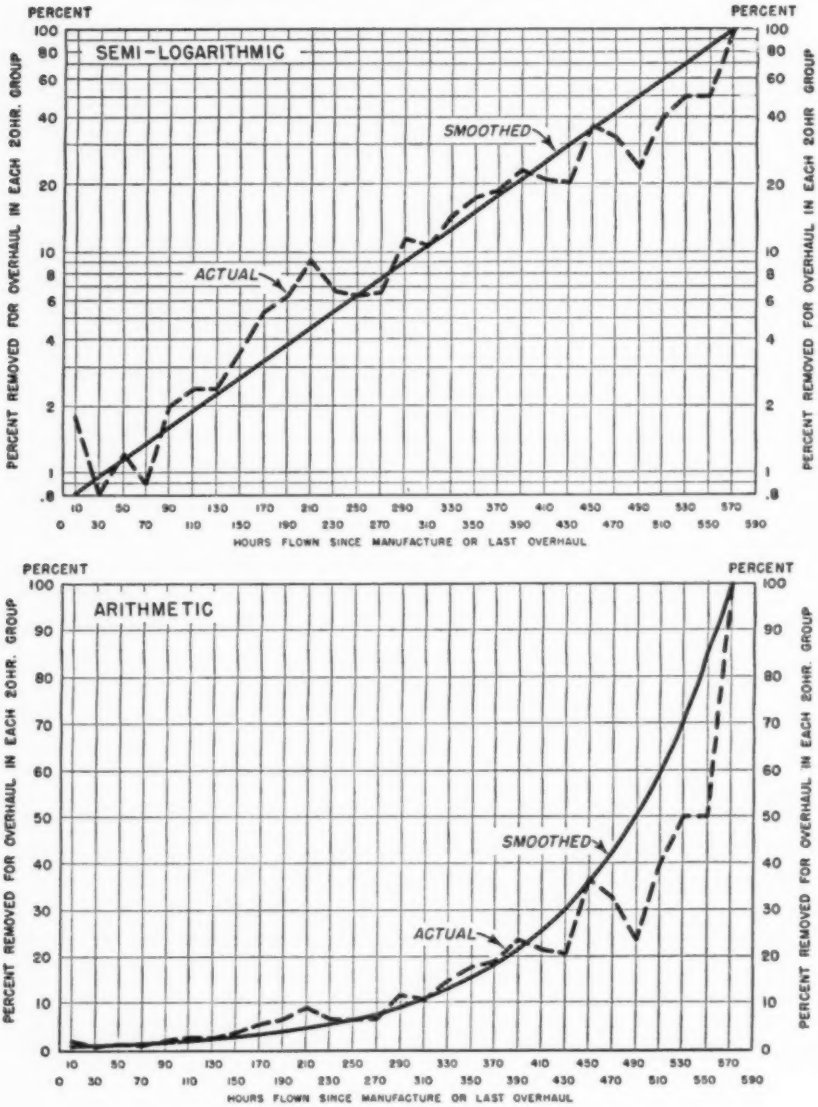
(4) In hour group X, a correction of $\frac{1}{2}$ is made for each engine in that hour group, except engines removed for "normal" engine failure, on the assumption that they are uniformly distributed through the hour group and are therefore exposed to failure for only $\frac{1}{2}$ the time.

(5) Each engine removed for "normal" engine failure in hour group X is counted as one because these removals during the hour group are considered as exposed to the end of the hour group.

For a more complete discussion of the general "exposed to risk" formulae, the reader is referred to H. S. Beers, "Notes on Exposure Formulae," *Transactions of the Actuarial Society of America*, Vol XLV, Page 41, and following discussion, May 1944.

CHART 1

OPERATING LIFE OF R-3350-23 ENGINES
ACTUAL AND SMOOTHED REMOVAL RATES
TRAINING OPERATIONS IN THE UNITED STATES, 1945



expectation of life." Specifically, this was accomplished for the end of any hour group X by (1) obtaining the sum of all survivorship per centages for all hour groups higher than X; (2) multiplying this sum by 20 hours to convert from the hour groups to the number of hours; (3) dividing this by the survivorship percentage of group X; and (4) adding one-half the hour interval to the result.

FINDINGS

Improvement in expectation of life of new R-3350 engines.—Sufficiently reliable information was collected to measure the effect of improvements made in successive models of the R-3350 engine upon the number of hours flown prior to their removal for first overhaul. Early combat operations in India, during the summer and fall of 1944, were conducted with B-29 aircraft equipped with unmodified R-3350-23A engines, which had an expectation of life of 163 hours prior to first overhaul. In February 1945, when the B-29 aircraft were equipped with -23A engines which had been modified and improved with more efficient cooling devices, the expectation of life of new engines rose to 280 hours, an increase of 117 hours per engine. The length of service was also increased substantially, as follows:

PER CENT OF NEW R-3350-23A (CARBURETOR TYPE) ENGINES SURVIVING TO SELECTED FLYING HOURS IN OPERATIONS FROM INDIA

Model	100 Hours	200 Hours	300 Hours	400 Hours
Unmodified	80.0	33.9	0.2	0
Modified	95.3	81.5	47.3	5.7

In February and March 1945, while the expectation of life prior to removal for first overhaul of the modified -23A engine was 280 hours in aircraft operating from India, it was 304 hours in aircraft operating from the Marianas, due largely to easier flying conditions in the islands. Later measurements indicated that the 304 hours would probably have been approximately 316 hours with more refined data.

The next model engine put into use was the fuel injection-equipped R-3350-57. Information is not available on the expectation of life of this engine prior to removal for the first overhaul in overseas operations. In the United States, however, the expectation of life of fuel injection engines was 329 hours compared with 310 hours for carburetor types engines. The increase in the expectation of life of 19 hours seems to indicate that the expectation of life of new fuel injection engines is some-

what better than of carburetor engines. The operating survival distribution was also lengthened somewhat, as follows:

PER CENT OF NEW R-3350 CARBURETOR AND FUEL INJECTION ENGINES SURVIVING TO SELECTED HOURS IN OPERATIONS IN THE UNITED STATES

Model	100 Hours	200 Hours	300 Hours	400 Hours	500 Hours
Carburetor (-23A)	94.1	81.7	57.9	24.6	2.4
Fuel Injection (-57)	95.0	84.3	63.3	31.2	4.9

It should be noted, however, that the fuel injection engines are safer and easier to operate, and the changeover was primariy planned for these reasons.

Expectation of life of new and overhauled engines.—The number of hours flown by new engines prior to removal for first overhaul is substantially greater than that flown by overhauled engines prior to second or later overhaul, despite the fact that during overhaul engines are completely disassembled, all worn parts replaced and the re-assembled engine thoroughly block-tested. Experience in the Marianas showed that the expectation of life was 316 hours for new engines and 270 hours for overhauled engines. Experience in training operations in the United States showed that the expectation of life was 310 hours for new, and 249 hours for overhauled engines. Thus the expectation of life of overhauled engines was 15 per cent lower in combat and 20 per cent lower in training than that of new engines.

The operating life of new engines before removal for overhaul was substantially longer than that of overhauled engines, both in the United States and overseas in the Marianas, as shown in the following table:

PER CENT OF NEW AND OVERHAULED R-3350-23A ENGINES SURVIVING TO SELECTED FLYING HOURS OVERSEAS AND IN THE UNITED STATES

Location and Overhaul Status	100 Hours	200 Hours	300 Hours	400 Hours	500 Hours
Overseas—combat					
New	96.8	87.5	62.7	19.0	0
Overhauled	92.5	75.5	43.4	8.8	0
United States—training					
New	94.1	81.7	57.9	24.6	2.4
Overhauled	87.0	65.4	36.4	10.4	0.5

It will be noticed that both new and overhauled modified carburetor engines in combat in the Marianas had greater expectations of life

than the respective groups in training in the United States. Ordinarily it would be expected that engines used in combat would not last as long as those in training. In this case, however, the situation was reversed. The very long range B-29's could climb very gradually after the take-off from the Marianas, thus saving the gasoline and avoiding the engine strain required to gain altitude quickly. In the United States, the airplanes were flown to high altitudes more quickly and, in addition, were probably put through extensive flying maneuvers as part of training operations. Furthermore, because of the extremely long distances to the target, the planes in the overseas group were flown "in combat" only a very small proportion of the total flight time to and from the target.

On the other hand, both the unmodified and the modified carburetor engines showed a much lower expectation of life in India than in the United States. The differences for new engines of both types are shown in the following table:

	Unmodified	*Modified*
India	163 hours	280 hours
United States	221 hours	310 hours

This was undoubtedly caused by the rugged flying conditions from India to enemy targets, which included rapid climb to get over the "Hump," and landing for refueling in China, followed by another take-off and another rapid climb to reach high altitudes over enemy held territory.

Use of expectation of life and survivorship percentages.—Requirements for spare engines to replace those removed for overhaul were determined by applying the estimated expectation of engine life to the planned schedule of hours to be flown. The planning factor for the unmodified R-3350-23A, adopted long before any B-29's were put in quantity operation, was 200 hours. Actual operations proved that this global planning rate was somewhat high. This rate resulted in an understatement of requirements in India, where the actual expectation of life was 163 hours. This understatement was compensated in part by an overstatement of requirements in the United States, where the actual proved to be 221 hours.

The global planning factor for the modified R-3350-23A, adopted long before these engines were put in quantity operation, was 300 hours. This rate proved to be remarkably accurate, and had the virtue of slightly overstating requirements for the Marianas and slightly understating those in the United States.

By the summer of 1945 it was clear that the planning rate of 300

hours would have to be revised downward for 1946 since the increasing proportion of overhauled to new engines would result in a lower expectation of life for all carburetor type engines, which would not be offset by the somewhat better performance of the fuel injection engines which were scheduled gradually to replace the carburetor engine.

For short-run supply planning and distribution for individual groups or areas, the distribution of survivorship percentages by 20 hour flying time groups (as shown in Table 2 and summarized in Chart 2), should be used instead of an expectation of life. The survivorship percentages

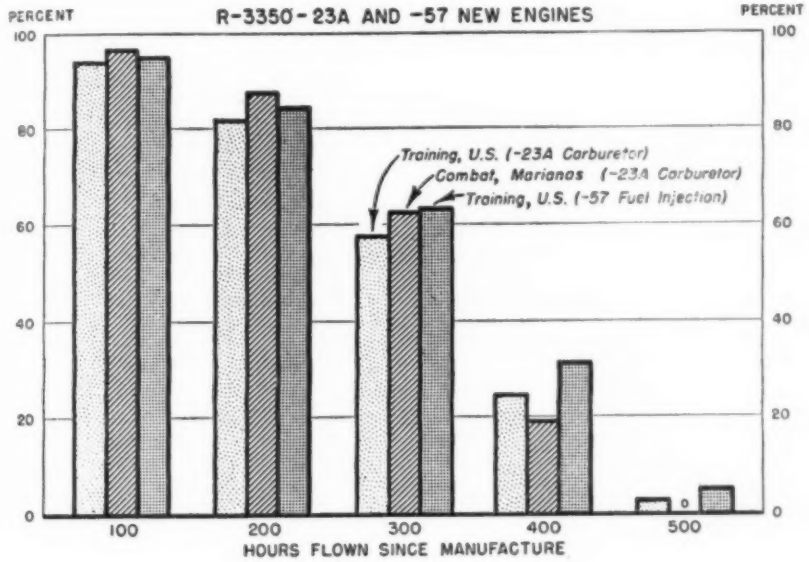
TABLE 2

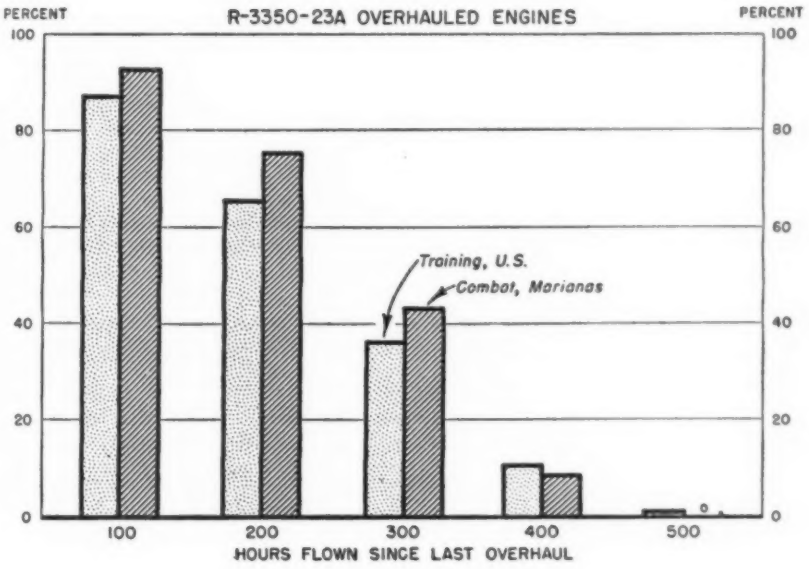
PER CENT OF ORIGINAL ENGINES SURVIVING AT EACH 20 HOURS
STUDY AS OF 31 JULY 1945

Number of Hours	Fuel Injection	Carburetor			
	Training, U. S.	Training, U. S.		Combat, Marianas	
	New	New	Overhauled	New	Overhauled
0	100.0	100.0	100.0	100.0	100.0
20	99.3	99.2	98.0	99.6	99.0
40	98.5	98.2	95.7	99.1	97.8
60	97.5	97.0	93.1	98.5	96.3
80	96.3	95.6	90.2	97.7	94.6
100	95.0	94.1	87.0	96.8	92.5
120	93.5	92.3	83.4	95.6	90.1
140	91.7	90.2	79.5	94.2	87.2
160	89.6	87.8	75.2	92.4	83.9
180	87.2	85.0	70.5	90.2	80.0
200	84.3	81.7	65.4	87.5	75.5
220	81.1	77.9	60.0	84.2	70.4
240	77.4	73.7	54.4	80.2	64.5
260	73.2	68.9	48.5	75.3	58.0
280	68.5	63.7	42.4	69.4	50.9
300	63.3	57.9	36.4	62.7	43.4
320	57.6	51.6	30.5	55.0	35.6
340	51.4	44.9	24.7	46.5	27.8
360	44.8	38.1	19.3	37.2	20.6
380	38.1	31.2	14.5	27.9	14.0
400	31.2	24.6	10.4	19.0	8.8
420	24.6	18.4	7.0	11.4	4.8
440	18.4	12.9	4.3	5.7	2.1
460	12.9	8.3	2.4	2.1	.7
480	8.4	4.8	1.2	.4	.1
500	4.9	2.4	.5	—	—
520	2.4	1.0	.2	—	—
540	1.0	.3	.1	—	—
560	.3	.1	—	—	—
580	.1	—	—	—	—

CHART 2

PER CENT OF ORIGINAL ENGINES SURVIVING TO EACH 100 HOURS BY STATUS AND ACTIVITY





should be converted into a table of probabilities of removal within the number of flying hours assigned for the period; and these probabilities should be applied to the flying time "age" distribution of the engines installed in aircraft at the beginning of the period to determine "normal" engine requirements.

TABLE 3
AVERAGE HOURS ON REMOVALS COMPARED WITH EXPECTATION OF LIFE OF R-3350-23A ENGINES IN COMBAT IN THE MARIANAS

Date	Expectation of Life	Average Hours on Removals
All engines		
30 Nov. 1944	—	91
31 Dec. 1944	—	133
31 Jan. 1945	—	151
20 Feb. 1945	271[1]	159
28 Feb. 1945	—	161
31 Mar. 1945	301	192
30 Apr. 1945[2]	310	234
31 May 1945	314	259
30 June 1945	308	268
31 July 1945	308	272
New Engines		
31 Mar. 1945	304	195
30 Apr. 1945[2]	310	236
31 May 1945	316	263
30 June 1945	316	273
31 July 1945	316	279
Overhauled engines		
31 Mar. 1945	289	136
30 Apr. 1945[2]	296	206
31 May 1945	294	215
30 June 1945	284	212
31 July 1945	270	206

[1] Losses of installed engines were omitted since data were not available for this calculation.
[2] Beginning 30 April 1945, removal rates were based on normal removals only. Removals directly attributable to accident or combat damage and engines removed for purposes of modification or model change were not used as removals.

Comparison of engine planning rates based on actuarial studies of all engines and studies of only removed engines.—Actual experience in the measurement and planning of R-3350 engine life prior to removal for overhaul demonstrated beyond doubt the superiority of the use of actuarial methods applied to the performance of both removed and installed engines over the use of averages based solely on the flying time of removed engines.

As was expected, the average number of hours flown by engines *removed for overhaul* increased steadily month after month. This upward

trend did not represent an increase in performance, however, but rather a change in the "age" distribution of installed engines. The average number of hours on engines removed from aircraft operating from the Marianas was 91 in November 1944, 161 in February 1945, and 272 in July 1945. (See Table 3.) Thus, even after nine months of operations, the average number of hours on removed engines had not yet attained the expectation of life of 316 hours to which it is asymptotic because the continuous and rapid increase of aircraft diluted the engine "age" distribution with new engines. Similar results were observed with respect to engines operating in the United States.

CONCLUSION

The use of actuarial methods to determine the expectation of life of engines under actual operating conditions is clearly superior to that of using averages based on the flying hours attained solely by engines removed for overhaul. This superiority is greatest when the engine "age" distribution or operating conditions are changing. The need for actuarial methods is most pressing when (1) costs are high and production, transportation or dollar resources are limited; (2) expectation of life of the engine is high, since it will then take many months, even under stable operating conditions, for the installed engine "age" distribution to become stabilized; and (3) the build-up of the total number of installed engines to its peak takes many months.

SYSTEMATIC SAMPLING AND ITS RELATION TO OTHER SAMPLING DESIGNS*

LILLIAN H. MADOW

This paper presents a simple introduction to the theory of systematic sampling. Its relation to random sampling, stratified random sampling and cluster sampling is shown by means of a numerical example. Some advice is given on the circumstances under which systematic sampling may be advantageously used.

IN DEVELOPING any given sample survey, many designs can yield sufficiently accurate estimates, if sufficiently large expenditures are made. The problem then is to select from among the various possible designs the most efficient one, i.e., the one that obtains the most information for a given expenditure of resources. The earliest statistical theory of sampling designs gave results for the unrestricted random design whereby any desired degree of accuracy could be obtained by altering the size of sample. In actually collecting data, however, the unrestricted random design was in general impractical administratively, and therefore rarely used. Alternative designs were used, some for which theory was available and some for which the estimates of error could only be guessed.

One basic sampling technique which has been in use for many years but for which until recently no supporting theory existed is the systematic sampling design, which, roughly speaking, consists in selecting every kth element of the population, the first element being selected at random. The systematic sample was often used in preference to other designs for which theory was available, because it was administratively simple and it was supposed that the results would be better than or at least close to those that would have been obtained by unrestricted random selection. It was also thought that the systematic design might take advantage of possible hidden stratification in the elements of the population. More refined considerations of the merits of the systematic design over the unrestricted random or other designs was hardly possible, since the theory of the systematic design was not known.

The beginnings of a theory of the systematic sampling design have already been given.[1] The results of that paper are here presented in less technical form and with numerical calculations carried out on il-

* Based on a thesis submitted to the Graduate Faculty of the American University in partial fulfillment of the requirements for the degree of Master of Arts, June 1944.

[1] "On the Theory of Systematic Sampling, I" by William G. Madow and Lillian H. Madow, in the *Annals of Mathematical Statistics*, March 1944.

lustrative examples. It is to be hoped that, with the theory of systematic sampling available to give the error of estimate of this design and to make effective comparisons with alternative designs, more confidence will be placed in a decision to use or not to use the systematic design in any given survey.

Illustrative example. In sampling a human population for a given characteristic, say to estimate the number of owner-occupied dwelling units in a given area, the procedure usually resolves itself into first dividing the area into strata by a geographic or rational grouping, and then estimating the characteristic within the strata by taking either an unrestricted random or a systematic sample. To compare these two designs in this situation, we shall assume that one of our strata consists of the 20 blocks of census tract S-6, Boston, Massachusetts, as enumerated in the 1940 housing census. This tract was selected only for its convenience in size. The data are presented in Table 1.

TABLE 1

CENSUS TRACT S-6, BOSTON, MASSACHUSETTS, APRIL 1, 1940

Block number	*Number of owner-occupied dwelling units* x_i	x_i^2
1	6	36
2	7	49
3	1	1
4	11	121
5	23	529
6	14	196
7	7	49
8	8	64
9	5	25
10	3	9
11	6	36
12	21	441
13	8	64
14	8	64
15	6	36
16	4	16
17	3	9
18	8	64
19	4	16
20	7	49
Σ	160	1874

Since the variance of the estimate of the mean of a sample drawn from a stratified population depends only on the within-strata variance

and not on the between-strata variance, any reduction in total variance must come from a reduction of the within-strata variance. For the data in Table 1, we shall investigate whether the unrestricted random or the systematic design gives the smallest within-strata variance. For this purpose, we can consider the Boston census tract a population P consisting of N elements, $x_1, x_2, \cdots x_N$, $N=20$. The mean value is given by

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i,$$

and the variance is

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N} (x_i - \bar{x})^2,$$

where by σ^2, the variance of the stratum which we are considering a subpopulation, is meant the variance of a random sample of one element from the population of N elements.

Substituting from Table 1 in the formula for $\bar{x}$ and σ^2, we see that

$$\bar{x} = \frac{160}{20} = 8.0 \quad \text{and} \quad \sigma^2 = \frac{1874}{20} - 64.0 = 29.7.$$

The problem now is to take for each proposed design a sample of n elements of the possible N, where $N=20$, to make an estimate, $\tilde{x}$, of the population mean, $\bar{x}$, where $\bar{x}=8.0$, and to find the variance of the estimate about the mean.

The unrestricted random sampling design. In this design, the procedure is to select at random n elements from the N possible ones, without replacement. The exact procedure may be something like the following. First, the elements of the population, if not already in some order, should be numbered from 1 to N. Then, if N is, say, 20, and it is desired to take a sample of 4 elements, a starting point in a 2-digit column of a table of random numbers is selected at random. If the random number is, say 16, then the element in the population numbered 16 is selected for the sample. If a number in the table is greater than 20, it may be skipped. This procedure is continued until 4 elements have been selected. If a second number 16 occurs in the table of random numbers, then it must be skipped, if the sampling design is to be without replacement. Other random procedures could also be defined.

For the design consisting of random sampling without replacement, the estimate of the population mean is given by

$$\tilde{x} = \frac{1}{n}\sum_{i=1}^{n} x_i,$$

where $\bar{x}$ is the arithmetic mean of the results of the n drawings, and x_i is the result of the ith drawing. This is an unbiased estimate of $\bar{x}$, and the variance of the estimate is given by

$$\sigma_{\bar{x}}^2 = \frac{N-n}{N-1}\frac{\sigma^2}{n},$$

where N, n, and σ^2 have already been defined.

For the example given in Table 1, suppose we consider the sampling procedure that consists of selecting 4 blocks at random from the set of 20 blocks. Then, in practice, we calculate $\bar{x}$ for the sample in order to estimate $\bar{x}$. Substituting in the above formula for $\sigma_{\bar{x}}^2$, we then have

$$\sigma_{\bar{x}}^2 = \frac{20-4}{20-1}\cdot\frac{29.7}{4} = 6.2526.$$

It is readily seen from the general formula, that the variance of this design depends only on the population variance, the size of n, and the relative sizes of n and N. To use the unrestricted random design effectively, then, it is necessary to have some rough idea of the variability of the elements in the population in order to estimate σ^2, and it is necessary to have a prelisting of the population so that a random design can be associated with the elements in the population. It is not necessary, however, to have any further knowledge about the characteristic of the population to be measured. It is not necessary to know whether the characteristic to be measured tends to cluster in groups or whether it is widely scattered throughout the population, or whether there is a serial trend in the characteristic. In collecting social or economic data, it would be unnecessary to know anything about the areal, color, or income level distribution of a population, if the design to be used is the unrestricted random one. The requirement of having a prelisting of the population that is up-to-date is one that is very often difficult to meet. If the prelisting is out of date, a random sample based on that prelisting will be biased.

The systematic sampling design. The theory of the systematic sampling design was developed directly in Reference 2, but it can also be derived in a simple manner from the theory of the cluster sampling design, since the systematic sampling design is cluster sampling in which the elements within clusters are arranged somewhat differently than usual.

In the usual type of cluster sampling, the procedure is to divide the population into classes or clusters, taking the elements of the popula-

tion in their natural order, i.e., the elements of each possible cluster are contiguous elements of the population. The sample then consists of several of the possible clusters, and there may be subsampling within clusters.

The theory of the systematic sampling design follows from the simplest case of cluster sampling, i.e., the case in which only one cluster is sampled and there is no subsampling within the cluster.[2] The big difference is that in the systematic design, if there are k possible clusters, the elements within any one of the clusters are k elements apart in the population, and no two elements within any of the possible clusters are contiguous elements of the population.

For the simplest cluster sampling design, the procedure is to divide the population of N elements into k clusters of n elements each. Only one cluster is taken in the sample and there is no subsampling within the cluster. In this case, each of the possible k clusters has the probability $1/k$ of being the one that is selected. $N=kn$.

For this design, we define $\bar{x}_i=\dfrac{\sum_{i=1}^{n} x_{ij}}{n}$, where x_{ij} is the jth element of the ith cluster, and $\bar{x}_i$ is the mean of the ith cluster, $i=1, \cdots, k$. The estimate of the population mean is given by $\tilde{x}$, where $\tilde{x}=\bar{x}_i$, if the ith possible sample is the one that is selected. This estimate is unbiased, and the variance is given by

$$\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n}[1 + (n-1)\rho],$$

where ρ is the intraclass correlation coefficient and is given by the equation

$$\rho\sigma^2 = \sigma_b^2 - \frac{\sigma_w^2}{n-1}.$$

In this expression, σ_b^2 is the variance between clusters and σ_w^2 is the variance within clusters, and they are defined by

$$\sigma_b^2 = \frac{\sum_{i=1}^{k} (\bar{x}_i - \bar{x})^2}{k} \quad \text{and} \quad \sigma_w^2 = \frac{\sum_{i=1}^{k} \sigma_i^2}{k},$$

[2] For detailed treatment of the cluster sampling design, see "On the Theory of Sampling from Finite Populations," by Morris H. Hansen and William N. Hurwitz, *Annals of Mathematical Statistics*, vol. 14, No. 4 (1943), pp. 333–362.

respectively. From the formula for $\sigma_{\bar{x}}^2$, it is now possible to see how much the variance depends on ρ, the intraclass correlation coefficient. It is easily shown that when $\rho = -1/N-1$, then the variance of the cluster sampling design is the same as that of the unrestricted random sampling design. Making this substitution for ρ, we have

$$\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n}\left[1 - \frac{n-1}{N-1}\right]$$
$$= \frac{\sigma^2}{n}\left[\frac{N-n}{N-1}\right],$$

the well-known expression for the variance of a random sample of n observations from N when the sampling is without replacement.

Moreover, it is readily observed that if ρ is less than $-1/N-1$, the variance of the cluster sampling design is less than that of the random sampling design, and if ρ is greater than $-1/N-1$, then the variance of the cluster sampling design is greater. Or, roughly speaking, if ρ is negative, cluster sampling yields a smaller variance than does unrestricted random sampling; if ρ is positive, the random sampling variance is less; and if ρ is approximately equal to 0, then the two designs yield approximately the same variance, although the variance of the cluster sampling design will be the greater one.

It is now possible to apply this formula to the Boston census tract we are using for illustration. Since we want $n=4$, the design will be to divide the population of 20 elements into 5 clusters of 4 elements each and to sample one of the clusters at random.

TABLE 2*

	C_1	C_2	C_3	C_4	C_5	
	6 7 1 11	23 14 7 8	5 3 6 21	8 8 6 4	3 8 4 7	
$\sum_{i=1}^{4} x_{ij}$	25	52	35	26	22	160
$\bar{x}_i$	6.25	13.0	8.75	6.5	5.5	
$\lvert\bar{x}_i - \bar{x}\rvert$	1.75	5.0	.75	1.5	2.5	
$(\bar{x}_i - \bar{x})^2$	3.0625	25.0	.5625	2.25	6.25	37.125

* The same data as in Table 1, but arranged in 5 clusters of 4 elements each.

Therefore,

$$\sigma_{\bar{x}}^2 = \frac{\sum_{i=1}^{5} (x_i - \bar{x})^2}{5} = \frac{37.125}{5} = 7.425.$$

To calculate ρ, we let

$$7.425 = \frac{29.7}{4}[1 + (4 - 1)\rho],$$

and we find that $\rho = 0$.

If we rearrange the data of the Boston census tract so that the elements within the possible clusters are 5 elements apart in the population, we have a systematic arrangement of the data. The basic calculations for this design are given in Table 3.

TABLE 3*

	K_1	K_2	K_3	K_4	K_5	
	6 14 6 4	7 7 21 3	1 8 8 8	11 5 8 4	23 3 6 7	
$\sum_{i=1}^{4} x_{ij}$	30	38	25	28	39	
$\bar{x}_i$	7.5	9.5	6.25	7.0	9.75	
$\lvert\bar{x}_i - \bar{x}\rvert$	.5	1.5	1.75	1.0	1.75	
$(\bar{x}_i - \bar{x})^2$	.25	2.25	3.0625	1.0	3.0625	9.625

* The data are the same as in Table 1, but arranged by taking 5 clusters of 4 elements each, the elements within each of the clusters being every 5th element of the population instead of being consecutively arranged within clusters. Contiguous elements of the population are to be found across clusters.

Based on the arrangement in Table 4, and using the formula for the variance of the cluster sampling design just given, we calculate

$$\sigma_{\bar{x}}^2 = \frac{9.625}{5} = 1.925.$$

To find ρ, we let $1.925 = \frac{29.7}{4}[1 + (4 - 1)\rho]$ and get $\rho = -.247$.

For this arrangement of the data, then, we see that there is a tremen-

dous gain in efficiency as a result of the systematic design, in this particular example.

In general, in the systematic design for selecting a sample of n elements from a population of $N = kn$ elements, the procedure for selecting the sample involves the following steps.

1. One of the numbers from 1 to k is selected at random.
2. If the number selected in Step 1 is i, then the ith element is selected for the sample.
3. Every kth element following the ith element is selected for the sample.

Let $\bar{x}$ now be the average of the n elements selected according to the above procedure. Then, $\bar{x}$ is unbiased, and has the variance

$$\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n}[1 + (n-1)\rho],$$

where ρ, the intraclass correlation coefficient has the same definition as before in cluster sampling. In this case, however, the clusters instead of having been, shall we say, given by nature, have been formed by the sampling procedure given above. There are k clusters, the ith cluster corresponding to the selection of the number i in Step 1 above.

To understand the character of the systematic design, i.e., to determine for which populations the systematic design is advantageous, it is desirable to substitute for the intraclass correlation coefficient a sum of serial correlation coefficients. In the cluster sampling design it is frequently too lengthy a job to calculate the intraclass correlation coefficient which is so important in estimating the necessary sample size. In many cases, however, it is possible from the nature of the characteristic of the population to be estimated to guess at a reasonably good figure for this intraclass correlation coefficient. At least, it is possible to determine whether it is expected to be positive, negative, or zero, just from the rational considerations of the problem, and an understanding of what ρ measures. Similarly, for the systematic design, it is useful to think in terms of the serial correlation coefficients, even though in a particular problem it may be too lengthy a job actually to perform the calculations. In the systematic sampling design, a good estimate of the variance depends on a good estimate of the serial correlation coefficients. These coefficients measure the degree of dependence of successive elements in the population.

The serial correlation coefficient, using the circular definition, is defined to be

$$\rho_{k\delta} = \frac{\sum_{\nu=1}^{kn} (x_\nu - \bar{x})(x_{\nu+\delta k} - \bar{x})}{kn\sigma^2}, \quad \text{where,}$$

if $\nu+\delta k > kn$, then its value is taken to be $\nu+\delta k-kn$, and if $\delta=n/2$, then

$$\rho_{k(n/2)} = \frac{\sum_{\nu=1}^{k(n/2)} (x_\nu - \bar{x})\left(x_{\nu+k(n/2)} - \bar{x}\right)}{kn\sigma^2}.$$

When we substitute for ρ in terms of the $\rho_{k\delta}$ in the formula for the variance of the systematic design, we obtain

$$\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n}\left[1 + 2\sum_{\delta} \rho_{k\delta}\right],$$

where $\delta=1, \cdots, n/2$, when n is even, and $\delta=1, \cdots, (n-1)/2$, when n is odd.

Comparisons between the efficiency of the systematic and random sampling designs can readily be made by observing that the variance of the systematic design is equal to that of the unrestricted random design, when

$$\sum_{\delta} \rho_{k\delta} = -\frac{n-1}{2(N-1)},$$

where $\delta=1, \cdots, n/2$ if n is even, and $\delta=1, \cdots, (n-1)/2$ if n is odd. If we substitute this expression for $\sum_{\delta}\rho_{k\delta}$ in the expression for

$$\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n}\left[1 + 2\sum_{\delta} \rho_{k\delta}\right],$$

where the limits for the summation with respect to δ are the same as those given above, then the expression reduces to

$$\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n}\left[\frac{N-n}{N-1}\right],$$

which is the well known expression for the variance of a random sample of n out of a finite population of N, when the sampling is without replacement. Moreover, it is readily observed that when the sum of the serial correlation coefficients is less than $-\frac{n-1}{2(N-1)}$, then the variance

of the systematic sampling design is less than that of the random sampling design, and if the sum of the serial correlation coefficients is greater than $-\frac{n-1}{2(N-1)}$, then the variance of the systematic sampling design is greater.

It is not possible to give simple directions for the conditions under which the systematic design is most appropriate. Some statements may, however, be helpful. Administratively, the systematic design is simple. Clearly, it is simpler to select every kth element from a card file, a map, or a prelisting, than to check with a table of random numbers before selecting each element to be included in the sample. The chief danger in applying the systematic design occurs when the data have a periodic formation, and the sampling interval chosen is equal to the period of the data. Past experience will provide useful information in the prediction of the relative efficiencies of different designs. When evidence is obtained that the systematic design is effective for one size of sample, care must be taken not to assume that it is effective for even a relatively close size of sample. As an illustration of the dangers of the uncritical use of the systematic design, an analysis of some data on seedlings is presented. In addition to testing the use of the systematic design, comparisons for several sample sizes are also made with the unrestricted random and the stratified random sampling designs.

Before giving the calculations for the seedling data, it might be well to use the Boston census tract data again to indicate the calculations that are made in connection with studying the applicability of the systematic design to a set of data. The calculations are too voluminous to reproduce for the seedling data, and therefore only the results will be given for them. For the Boston census tract, however, it is possible to give all the calculations.

The basic data and calculations are given in Table 4.

We have already calculated for these data that $\bar{x}=8.0$ and $\sigma^2=29.7$. From Table 5 we can now calculate

$$\sigma^2\rho_{5,1} = \frac{1101}{20} - 64.0 = -8.95; \qquad \therefore \quad \rho_{5,1} = -.30135, \quad \text{and}$$

$$\frac{1}{2}(\sigma^2\rho_{5,2}) = \frac{1}{2}\left(\frac{1198}{20} - 64.0\right) = -2.05;$$

$$\therefore \frac{1}{2}\rho_{5,2} = -.06902 \qquad \therefore \sum_{\delta=1}^{n/2} \rho_{k\delta} = -.37037$$

Therefore, $\sigma_{\bar{x}}^2 = \frac{29.7}{4}[1+2(-.37037)] = 1.925$. This is the same figure that was obtained for the cluster design calculated on the basis of Table 4 when the elements were arranged in systematic order and only one

TABLE 4*

Block number	*Number of owner-occupied dwelling units* x_i	$x_i x_{i+1}$	$x_i x_{i+10}$	x_i^2
1	6	6·14 = 84	6·6 = 36	36
2	7	7·7 = 49	7·21 = 147	49
3	1	1·8 = 8	1·8 = 8	1
4	11	11·5 = 55	11·8 = 88	121
5	23	23·3 = 69	23·6 = 138	529
6	14	14·6 = 84	14·4 = 56	196
7	7	7·21 = 147	7·3 = 21	49
8	8	8·8 = 64	8·8 = 64	64
9	5	5·8 = 40	5·4 = 20	25
10	3	3·6 = 18	3·7 = 21	9
11	6	6·4 = 24	6·6 = 36	36
12	21	21·3 = 63	21·7 = 147	441
13	8	8·8 = 64	8·1 = 8	64
14	8	8·4 = 32	8·11 = 88	64
15	6	6·7 = 42	6·23 = 138	36
16	4	4·6 = 24	4·14 = 56	16
17	3	3·7 = 21	3·7 = 21	9
18	8	8·1 = 8	8·8 = 64	64
19	4	4·11 = 44	4·5 = 20	16
20	7	7·23 = 161	7·3 = 21	49
$\sum_{i=1}^{20}$	160	1101	1198	1874

* The same data as given in Table 1, with some additional columns of computation.

cluster of 4 elements was chosen in the sample. This agreement is in accordance with the theory.

Data on seedlings. Calculations of the same type were carried out on some seedling data collected by Floyd A. Johnson, Iowa State College. The data consist of 420 observations on seedlings in one bed of a forest nursery. The bed had 4 rows and was 420 feet long. Each observation represents one foot of bed-width, or the total of the four rows in that particular foot of bed. Some of the results are given in Table 5.

TABLE 5

n	k	$2\sum_{\delta}\rho_{k\delta}$*	σ_x^2
10	42	−.2997	4.213
21	20	−.1945	3.061
28	15	−.1781	2.148
30	14	−.4012	.693
42	10	−.1521	1.742
60	7	−.4250	.263
84	5	−.0129	1.220

* $\rho_{k\delta}$ is summed from $\delta=1$ to $\delta=n/2$ if n is even or to $n-1/2$ if n is odd.

For the same values of n as given in Table 5, calculations were made for the variance of the unrestricted random and the stratified random designs. The stratified random design was simple proportionate sampling within strata. The strata were of equal size and one element was selected from each stratum. The results are given in Table 6 together with a comparison of the relative efficiencies of the systematic and stratified random designs with respect to the unrestricted random design.

TABLE 6

n	σ_R^2	σ_{St}^2	σ_{Sys}^2	Eff_{St}	Eff_{Sys}
1	2	3	4	5	6
10	10.293	7.212	4.213	42.7	144.3
21	4.770	2.999	3.061	59.1	55.8
28	3.515	2.092	2.418	68.0	45.4
30	3.264	1.902	.693	71.6	371.0
42	2.259	1.292	1.742	74.8	29.7
60	1.506	.8183	.2631	84.0	472.4
84	1.004	.5052	1.220	98.7	−17.7

In Table 6, the first column represents the sample size, and columns 2, 3, and 4 represent the variances of the unrestricted random, the stratified random, and the systematic sampling designs, respectively. Columns 5 and 6 represent the relative gain in efficiency of the stratified random and systematic designs, respectively, over the unrestricted random design. The efficiency of the alternative designs was measured by dividing the variance of the unrestricted random design by that of the restricted design, and expressing the result as a per cent. By this measure, a restricted design has the same efficiency as the unrestricted random design, if the measure is 100 per cent. If the restricted design is more efficient than the unrestricted random design, then the measure will be greater than 100 per cent, and if the restricted design

is less efficient, then the measure will be less than 100 per cent. This measure of relative efficiency cannot be negative, of course, by definition.

Another way of looking at this measure of relative efficiency is in terms of sample size. For example, for $n=10$, Table 6 shows that for every element in a stratified sample, the unrestricted random sample requires about one and one-half elements to have the same efficiency. Similarly, for $n=10$, the random sample requires about two and one-half elements for every element of the systematic design to have the same efficiency.

Table 5 shows some interesting information about some characteristics of the systematic design. First, we notice that the value of $\sum_{\delta}\rho_{k\delta}$, although always negative, is extremely variable, and that it does not vary consistently with either n or k. The value of $\sum_{\delta}\rho_{k\delta}$, for any given value of n or k, depends not so much on the value of n or k as on the arrangement of the elements in the original population and the degree to which successive elements in the population are positively or negatively correlated. We also see that although, in general, the variance of the systematic design decreases with increasing size of n, this decrease is not uniform. For example, the variance at $n=30$ is much lower than that at $n=28$ or $n=42$. Again, the variance at $n=60$ is much lower than at $n=42$ or $n=84$.

From Table 6, we can compare the variances of the systematic design with two other designs for the same sample sizes. The variance of the systematic design for different values of n fluctuates, whereas the variances of the proportionate stratified random and unrestricted random designs behave in a stable manner. For the latter designs it can always be said that if the variance is known for a given n, the variance for a sample of $n+m$ will be smaller, no matter what the value of m. This statement does not hold for the systematic design.

The stratified random design shows a steady increase in efficiency over the unrestricted design for increasing values of n, as can be seen from Table 6. The efficiency of the systematic design as compared with the unrestricted random design, however, is erratic. The systematic design is more efficient than the stratified random design in the cases when $n=10$, 30, or 60, but it is less efficient than the stratified random design for the cases in which $n=21$, 28, 42, or 84. In all cases except for $n=84$, the systematic design is more efficient than the unrestricted random design, but for $n=84$ the relative efficiency is less than 100, being only 83.3 per cent.

For these data, the stratified random is more efficient than the unrestricted random design for all values of n, but it never attains the degree of efficiency that the systematic design attains. In the cases where the systematic design is more efficient than the stratified random design, the systematic design is about twice as efficient as the stratified random design, whereas in most of the cases in which the systematic design is less efficient than the stratified random design, the stratified design has only a slight gain over the systematic design.

CALCULATING THE GEOMETRIC MEAN FROM A LARGE AMOUNT OF DATA

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This note presents a short procedure for calculating the geometric mean from a large amount of data. The method involves a) using a grouping of the frequency distribution such that the ratios of the class limits are equal, and b) an application of the "short method" for calculating the arithmetic average. By this procedure the calculation of the geometric mean takes more time than that of the arithmetic mean, only to the extent of getting the logarithms of four values and the antilogarithm of one value. Analogous procedure can be used for the harmonic and other means.

INTRODUCTION. Discussions in statistics texts leave the impression that the calculation of the geometric mean of a large number of measurements requires much more time than the computation of the arithmetic average. The procedures usually presented involve either a) the determination of the logarithms of each measurement, which is unnecessary; or b) the use of mid-points of equal intervals of the arithmetic values, which gives the resulting geometric mean an upward bias. The purpose of this note is to show that it is possible to calculate the geometric mean of a large number of measurements just about as easily as the arithmetic average. By this procedure the calculation of the geometric mean takes more time than that of the arithmetic mean, only to the extent of getting the logarithms of four values and the antilogarithm of one value. Furthermore, analogous procedure can be used in obtaining the harmonic and other means, which by definition, involve arithmetic averages of some functions of the original values. Grouping the frequency distribution with suitably chosen unequal intervals and the application of the so-called "short method" for calculating the arithmetic average are the basis of the procedure to be demonstrated.

The General Idea. The geometric and harmonic means are examples of a more general averaging process which may be described as follows: X, the original variable, is transformed by means of a relationship $Z=f(X)$. (For the geometric mean, $Z=\log X$). Then the arithmetic average of the Z's, $\overline{Z}$, is obtained and the desired average of the X's, $\hat{X}$, is merely the value of X corresponding to $\overline{Z}$. (For the geometric average, $\hat{X}=\text{antilog } \overline{Z}$. The idea of the proposed procedure is that, if a grouping method with equal class intervals on the Z scale is used,

then it is not necessary to apply the transformation $Z=f(X)$, for each measurement. It is sufficient to find the X values corresponding to the class limits on the Z scale. This yields unequal intervals on the X scale according to which the X values can be grouped.

Demonstration of Method. In the following, a short practical way of calculating the geometric average for a large amount of data is explained using the data in Table 1.

TABLE 1
THE DATA

515.8	354.7	291.3	306.7	237.3
252.6	223.8	279.2	370.0	364.6
301.0	334.3	349.0	390.7	264.3
288.9	318.8	353.7	417.1	356.5
386.6	189.0*	482.9	218.5	343.9
584.8	269.8	398.6	412.5	415.3
354.3	295.3	554.4	331.5	431.3
592.0*	241.3	373.3	402.2	295.7
373.7	472.4	302.4	406.7	260.5
331.4	245.0	354.6	358.0	282.1

* High and low values.

By definition

$$\log G = \frac{\sum f \log X}{N},$$

where G is the geometric mean, X is the value of the measurement, f its frequency, and N is the total frequency. This formula can be used if the frequency distribution of the logarithms is known. It is possible to get this distribution in a grouped form without getting the logarithms of the data.

Step #1. The Class Intervals. In order to avoid getting logarithms of each value of the data, we determine class intervals such that the ratios of the upper to the lower limit are constant. These are appropriate because then the corresponding grouping of the logarithms would have equal intervals. For the data in Table 1,

(High value)	Log 592.0 =	2.7723
(Low value)	Log 189.0 =	2.2765
	Range of logarithms =	.4958

Hence, in order to have seven classes, the range of the class interval of the logarithms should be about .071. Since the value .071 corresponds approximately to an antilogarithm of 1.18, 1.2 would be a

convenient ratio to use in determining the grouping (with unequal intervals) of the arithmetic values. Then the class intervals for the logarithms would have a range of log 1.2 = .0792. Thus, arbitrarily using 180 as the lower limit of the first class, we get the class intervals given in Table 2. These intervals are obtained by multiplying 180 times 1.2 to get 216, 216 by 1.2 to get 259, etc. (A slide rule is adequate.)

TABLE 2
THE FREQUENCY DISTRIBUTION

Class intervals*	Frequency f	d	fd
180–216	1	−3	− 3
216–259	6	−2	−12
259–311	12	−1	−12
311–373	14	0	0
373–448	11	1	11
448–537	3	2	6
537–645	3	3	9
Sums	50	0	− 2

* Lower limits included in intervals.

Step # 2. The Frequency Distribution. The grouping of the data according to the unequal intervals in Table 2 is made in the usual way, and involves no more work than if the intervals were equal. It is important to see that this is also the grouping of the logarithms where the equal class intervals have a range of .0792.

Step #3. Calculation of the Geometric Mean. The so-called "short method" for calculating an arithmetic average from a frequency distribution can be applied in calculating the arithmetic average of the logarithms whose frequency distribution is given in Table 2. In this case the assumed mean is the mid-point of the class, log 311 − log 373, or 2.5323. Hence,

$$\log G = 2.5323 + .0792\left(-\frac{2}{50}\right) = 2.5291, \quad \text{or}$$

$$G = 338.2.$$

The above involves an application of the formulas

$$\overline{X} = \overline{X}' + i\bar{d}, \quad d = \frac{X - \overline{X}'}{i},$$

where $\overline{X}'$ is the value of the assumed mean, $\bar{d}$ the arithmetic average of the d's, and i is the range of the class interval.

THE DESIGN AND ANALYSIS OF METHODS FOR SAMPLING MICROCLIMATIC FACTORS

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The purpose of this paper is to outline the principles and use of a method for sampling factors—such as the amount of rain reaching the ground under a forest during a series of storms—which exhibit variations in both space and time. Solutions of two somewhat different sampling problems are discussed. One is designed simply to provide a sample estimate (average or sum) for any desired factor over a period of time. The other, with little loss of efficiency, provides a sample estimate for each of a series of short time units making up the period, as well as an estimate for the period as a whole. It is shown that these sampling methods are likely to supply a substantial gain in efficiency as compared to methods that have been commonly employed.

IN RECENT years the intensive study of microclimate has added a new task to the science of climatology: that of sampling and comparing the average magnitudes of microclimatic factors on relatively small land areas. As illustrations, the climatologist may wish to compare the average amounts of evaporation and transpiration of water from fields containing different crops in a formally designed experiment, or the amounts of rainfall reaching the ground on watersheds or plots containing different kinds or densities of plant cover. In such studies the use of a single climatological station in each field or watershed fails to provide records adequate for reliable comparisons, and it becomes necessary to sample the desired factor or factors intensively in order to minimize errors associated with local variation.

By contrast, much of the earlier climatic research—like that in other fields—was designed to find out facts about the behavior of various climatic factors over a period of time; usually through the use of single climatological stations maintained at selected positions in space. With the aid of inductive reasoning, repeated experiments of this sort have contributed greatly to our fund of knowledge on the general behavior of these factors. When viewed in the light of our newer requirements, however, such studies are inadequate for deriving quantitative esti-

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mates of the average magnitude of any factor on a land area large enough to contain local variations in microclimate.

The solution of this newer problem is not difficult; it is only necessary to sample each area by some standard technique, with continuous records obtained from a number of climatological instruments or stations installed at randomized points in the area so as to reduce sampling errors. This method may, however, be extravagant in cost, since it calls for a number of expensive instruments requiring considerable attention, and provides unnecessarily precise information on time variations while adequately minimizing the sampling errors which are associated with variations in space. Sometimes, in fact, the necessary number of instruments may incur prohibitive costs. One writer has even stated[2] that "it was early discovered that rain in the forest cannot be measured" because of excessive local variations in net precipitation under a forest canopy.

The purpose of this article is to outline a more efficient method for sampling microclimate and to discuss the statistical methods of analysis that are suited to it. With appropriate modifications, this method should be adaptable also to the sampling of other variables that are characterized by variations in both time and space. In brief, it requires the use of only a small number of instruments for each area to be sampled; these instruments are not placed at fixed positions in the area for the duration of the study, but are moved after each sampling observation to new randomized positions. Each individual observation contains sampling errors associated with both the time and the place at which it is taken. With properly designed sampling, however, unconfounded average or total values can be obtained for any desired time period long enough to permit thorough sampling in both time and space, and both types of sampling error can be simultaneously minimized with a good deal of efficiency. If these moving instruments are supplemented by one or more fixed stations—employed for the statistical control of variations associated only with time—, even short-time (as daily) average values may generally be calculated with satisfactory precision and little additional field work.

This method has been used in sampling net rainfall under a conifer forest[3,4] with a striking increase in efficiency as compared to the use of rain-gages kept at fixed locations. In a randomized-block experiment

[2] Rudolf Geiger, "Das Klima der bodennahen Luftschicht," *Die Wissenschaft*, vol. 78 (1927).

[3] C. H. Niederhof and H. G. Wilm, Effect of cutting mature lodgepole pine on rainfall interception, *Journal of Forestry*, vol. 41, pp. 57–61. 1943.

[4] H. G. Wilm, Determining net rainfall under a conifer forest, *Journal of Agricultural Research* vol. 67, no. 12, pp. 501–512. 1943.

a total of 80 gages, moved after each of 24 storms to sample new locations on each of 20 plots, provided data on average net and gross rainfall that were as precise as though over 400 fixed gages had been employed.

PRINCIPLES OF THE SAMPLING METHOD

Any factor to be sampled may be thought of as characterized by a true value μ for a given area and season or other relatively long period of time, and by variations within the area and period due to two general sources: a component η due to the time unit (as a day or storm) during which any single sample observation is taken, and a second component ϵ associated with the position of the sample observation in the area at that time. Then the magnitude of a single observation Y, taken during the ith time unit of the period and at the jth position in the area, may be expressed as:

$$Y_{ij} = \mu + \eta_i + \epsilon_{ij}. \qquad \text{I}$$

As stated in this equation, the magnitude of the time component is common to all observations taken in the area during any particular time unit, while the component of position is peculiar to each observation obtained during that time unit. If both are random and independent in the probability sense, the mean value of each or their joint value will tend toward zero with increasing numbers of observations.

In most investigations it is not practical to make an enumeration of all possible positions and time units in an area and period of time; instead it is necessary to take a randomized sample of these two populations. From such sampling the true average value μ and the true variance σ^2 of either population cannot be determined exactly; it is possible to obtain only a sample average and an estimate of the error of this average in approximating μ. If $Y_1, Y_2, \cdots Y_n$ are n values of a random variable drawn from a population whose true average and variance are μ and σ^2, then ($\bar{y}$ being the average of the observed values),[5]

$$\frac{S(Y - \bar{y})^2}{n - 1} = s^2, \text{ which provides an estimate of } \sigma^2. \qquad \text{II}$$

If the sample is made up of n observations, the variance of their average is equal to s^2/n; the variance of their sum is ns^2; and the square root of either value (standard error of the sample average or sum) provides an estimate of the failure of the sample statistic to equal the population value.

[5] For the symbolism employed in this paper, see G. W. Snedecor, *Statistical Methods*, 4th edition (pp. 34–35). Iowa State College Press, 1946.

In the sampling problem under discussion there are two categories of variation (time and space), each of which may be described by a variance. Then the variance of a single observation Y, randomized in both time and space, may be expressed as

$$s_Y^2 = s_t^2 + s_s^2 \qquad \text{III}$$

and the variance of the average of sample observations taken at k new places during each of n time units is

$$s_{\bar{y}}^2 = s_t^2/n + s_s^2/kn; \qquad \text{IV}$$

that is, the variance of the average is reduced in proportion to the number of observations taken in each category of variation.[6] If the sampling observations in time are taken repeatedly at the same set of k positions, then

$$s_{\bar{y}}^2 = s_t^2/n + s_s^2/k. \qquad \text{V}$$

And if a single instrument is left continuously at the same position (as in earlier studies), the variance s_t^2/n becomes zero, while s_s^2 is not at all reduced. If the position variance is at all large, this procedure may cause a serious error in estimating the population value, simply because the available data describe only one point in the area sampled. It is evident that the sampling error of an average is likely to be minimized with greatest efficiency by taking observations at new places during each of a number of time units (equation IV).

Up to this point our discussion has dealt with random sampling in both time and space: a type of sampling which may be employed in the study of microclimate, although continuous observations in time may be more frequently desirable. In sampling microclimatic factors it is ordinarily necessary to visit each sampling point twice in order to obtain a single observation: once to install the instrument, and once to obtain the observation at the end of a short unit of time. For this reason and because values for short time units are frequently desired, it is usually most efficient to move each instrument to a new sampling location immediately after each reading is taken, so that the resulting observations follow each other without a break. By this means a complete enumeration is obtained of variations in time. Then, introducing the correction for a finite population, equation IV becomes

$$s_{\bar{y}}^2 = \frac{s_t^2}{n}\left\{\frac{N-n}{n}\right\} + \frac{s_s^2}{kn}\left\{\frac{K-kn}{K}\right\} \qquad \text{VI}$$

[6] Neglecting corrections for finite populations, which adjust the error variance of means to zero as n and k become equal to the whole population. (See F. X. Schumacher and R. A. Chapman, "Sampling Methods in Forestry and Range Management," *Duke University School of Forestry Bulletin* 7. 1942.)

where N and K are the total numbers of time units and positions in the two populations; and since $n=N$ when all time units are enumerated, the first right-hand term becomes zero.

DESIGN AND ANALYSIS OF THE SAMPLES

In the following sections are discussed the design and analysis of two variations of the general sampling method, as developed from the above principles. For the sake of clarity, the discussion includes only the simplest sampling designs; these may be varied and amplified to suit a variety of requirements. In the first sampling problem, the objective is to obtain a reliable average value for any desired factor on a land area over a period of time such as a month or growing season. In the second, the aim is to obtain relatively precise average values for the area on successive days or other short time units throughout the whole period as well as an average value for the period.

First problem: the estimation of an average value for a period of time.—If the investigator desires simply an average value for any factor, based on consecutive observations taken over a period of time such as a growing season, the only requirement is to obtain an unbiased estimate of this statistic and of its sampling error in space (as estimated by s_ϵ^2). These may be obtained by taking sample observations at two or more new positions, randomized over the area to be sampled, during each of a number of consecutive time units (as days) which add up to the whole period. Then the variations in time and space which have been sampled by this procedure may be partitioned into their components by a simple analysis of variance (Table 1).

TABLE 1

ANALYSIS OF VARIANCE IN THE SAMPLING OF MICROCLIMATE

Source of variation	*Degrees of freedom*	*Variances estimated by each mean square*
(1) Total	$kn-1$	
(2) Time units	$n-1$	$ks_{\bar{y}}^2+s_\epsilon^2$
(3) Observations within time units	$n(k-1)$	s_ϵ^2

Since the sampling error variance of the average value obtained by this method contains only s_ϵ^2, for our present purpose it is necessary to calculate only the mean square for "observations within time units" in Table 1. From this mean square the standard error of the sample average may be calculated:

$$s_{\bar{y}} = \pm \sqrt{\frac{s_\epsilon^2}{kn}\left\{\frac{K-kn}{K}\right\}}. \qquad \text{VII}$$

This sampling method is illustrated by data obtained in an investigation of the quantity of net rainfall reaching the ground under a forest of young lodgepole pine (Table 2, "Net Rainfall": columns headed "Y"). In this study two standard rain-gages were placed at newly

TABLE 2

GROSS AND NET RAINFALL IN A FOREST OF YOUNG LODGEPOLE PINE

Storm No.	Rainfall—inches			Storm No.	Rainfall—inches		
	Gross	*Net*	*Calculated average*		*Gross*	*Net*	*Calculated average*
	(X)	(Y)	(E)		(X)	(Y)	(E)
1	0.09	0.08 .08	0.060	11	0.14	0.06 .12	0.100
2	1.23	1.01 .72	.974	12	.30	.28 .30	.228
3	.92	.76 .71	.725	13	.72	.62 .95	.565
4	.22	.20 .14	.164	14	.20	.11 .15	.148
5	.08	.05 .04	.052	15	.14	.06 .07	.100
6	.11	.09 .07	.076	16	.24	.08 .08	.180
7	.11	.04 .08	.076	17	.27	.28 .18	.204
8	.53	.38 .44	.413	18	.17	.16 .16	.124
9	.06	.03 .06	.036	19	.09	.06 .04	.060
10	.46	.26 .36	.356	20	.24	.12 .16	.180
Average total rainfall					6.32	4.82	
Average rainfall per storm					0.316	0.241	

Sum of squares "within storms" = 0.1167
mean square = 0.0058

randomized positions in the forest during each of 20 successive storms, so that at the end of the study period a total of 40 positions had been sampled. The observed sample value for the total net rainfall over the 20 storms was 4.82 inches of water; and the average net rainfall per storm was 0.241 inch. From equation VII, the standard error of this

average in estimating the true value for the area and period was calculated to be ± 0.012 inch.[7]

The results of another study of net rainfall can also be cited to illustrate the precision of our short-cut method. In this experiment rainfall data were obtained at 20 positions in a pine forest, with the gages left at the same positions for a period of two years. Since the gage positions were not properly randomized, the resulting data may not provide unbiased estimates of sampling errors; but they can serve to show the benefits of more efficient sampling. For this purpose a consecutive series of 10 storms was selected, all occurring in a single month. Based on these 200 observations, the average net rainfall for these storms was 0.2973 inch; the smallest and largest storms averaged 0.009 and 1.036 inch; and the range from the smallest to the largest reading in a single storm was 0.58 inch. From these data 4 different samples of 20 readings were drawn, each composed of 2 readings taken at random without replacement from the 20 readings available for each of the 10 storms. The resulting sample averages were 0.3000, 0.3045, 0.3085, and 0.2950 inch. Thus even the poorest of these small samples provided an average value which deviated only 0.011 inch (3.8 per cent) from that provided by 10 times as many observations obtained with gages at fixed locations.

Second problem: the estimation of average values for each of a series of short-time units.—The relatively simple procedure required for solution of the first problem becomes somewhat more complex if a reliable average value is desired for each short-time unit in the whole period. Returning to equations I and III, we have seen that any single observation contains sampling errors due to the particular time unit and position at which it is taken; and that each of these components of error can be estimated by a variance. Hence it can be appreciated that, even though the average of a series of randomized observations may be quite precise (equations IV and VI), each separate observation or the average of only those obtained during a single time unit provides a relatively poor estimate of the desired factor for the whole area during that time unit. This fact is illustrated by the variation between the members of each pair of observations in Table 2, and by the large standard error of their average (pooled standard error $= \pm 0.054$ inch).

In order to obtain a more reliable average value for each time unit, we might take a considerable number of randomized observations during each of the series of units that make up the whole time period. But,

[7] Since variations in net rainfall tend to be correlated with the amount of rainfall in the storm, the logarithms of the data from this experiment might preferably have been employed in analysis. The natural numbers were used, however, in order to simplify the discussion of sampling principles.

as in the first problem, this procedure is unwieldy and may be made more effective. In order to increase efficiency we can proceed exactly as in the first problem, taking only two (or k) randomized observations during each time unit. For our new purpose, however—the statistical control of time variations—, one fixed station (or more than one, if desired) is employed in addition to the two moving instruments, with the new station maintained continuously at some convenient point in the area sampled. In order to minimize the chance of bias, this station should be carefully located and care should be taken to avoid the trampling of crops or any other influence that might be cumulative and peculiar to the fixed station.[8] Observations taken at the fixed station during successive time units will completely measure variations with time at this point; they will fail to do so for the whole area only to the extent that the true value for the whole area during each time unit interacts with the corresponding value observed at the fixed station. With the aid of regression analysis, these data will provide a basis for estimating the most probable average value for the space population during each of the time units which make up the period of study.

For this purpose a regression equation can be calculated to express the relation of observations taken at the moving stations (Y) to those taken at the fixed station. This regression should ordinarily be linear and quite strong; if it is linear it may assume the form

$$E = \bar{y} + b(X - \bar{x}) \qquad \text{VIII}$$

where E is an average value calculated for the space population during any single time unit; b is the linear regression coefficient computed from the corrected squares and products for "time units" (line 2 in Table 1); X is any single observation taken at the fixed station; and $\bar{x}$ is the general mean of all fixed-station observations.

From equation VIII can be computed an average value E for each time unit, corresponding to each value X observed at the fixed station. And the sampling variance of E may be calculated from an equation of the form

$$V_E = V_{y \cdot x}\left\{\frac{1}{kn} + \frac{(X - \bar{x})^2}{Sx^2}\right\} \qquad \text{IX}$$

where V_E is the desired error variance; $V_{y \cdot x}$ is the mean square for errors of estimate derived from the regression analysis (in line 2, Table 1); and Sx^2 is the corrected sum of squares for the fixed-station data. This

[8] Any such bias will not affect the average values obtained from the sample; but it might affect values calculated for individual time units, and it will be reflected in the magnitudes of the sampling errors for time-unit averages.

equation estimates the error of any value E in estimating the true average value for the population in space and time which is characterized by that single value for X from which the value E was calculated. As indicated by the right-hand term of the equation, the precision of the estimate E decreases as the deviation of X from the average value ($\bar{x}$) increases; where X is close to the average, E may be almost as precise as the average of all the moving-station data. When, on the other hand, X is very small or large, the calculated value E may be relatively unprecise. Hence this sampling method may not give satisfactory results where it is desired to obtain reliable estimates for extreme conditions.

It should be observed that, although the regression may have removed most of the effects of time variation on average values and their errors, the quantity $V_{y.x}$ may still contain variances associated with both time and space, as shown in line 2 of Table 1; but now the time variance estimates only a residual effect: the failure of the fixed station to isolate completely variations with time on the whole sampled area. Thus the mean square for errors of estimate may be expressed as

$$V_{y \cdot x} = k s_{y'}^2 + s_\epsilon^2 \qquad \text{X}$$

where $s_{y'}^2$ is the residual time variance. Incidentally, any bias associated with the fixed station should be expressed in the magnitude of this residual variance.

As it is developed above, equation IX is not entirely adequate for calculating the error of short-time values unless a single time unit and sampling position may be considered a minute portion of the populations in time and space which are being sampled. Ordinarily even the mean of a considerable number of sample observations in space may be so considered, as a single rain-gage or thermograph occupies only an infinitesimal part of even a single acre of land. In our sampling problem, however, the time population is completely enumerated; and even the residual error estimated by the variance $s_{y'}^2$ for a single time unit may represent an appreciable portion of the population of these time deviations. In the calculation of V_E from equation IX, therefore, it may be desirable to break down $V_{y.x}$ into its two components, and to adjust the time component for its sampling of a finite population. Then equation IX assumes the modified form

$$V_E = \left[k s_{y'}^2 \left\{ \frac{n-1}{n} \right\} + s_\epsilon^2 \right] \left[\frac{1}{kn} + \frac{(X - \bar{x})^2}{Sx^2} \right]. \qquad \text{XI}$$

Obviously the use of this more complex error equation may induce

little gain in precision unless $s_{\eta'}^2$ forms a substantial part of the mean square for errors of estimate.

When the error variance of a seasonal average is calculated, all the right-hand terms of equation XI except s_ϵ^2/kn become zero; as in our first problem, the average contains only sampling errors associated with variations in space.

This extension of the simpler method of sampling may again be illustrated by data obtained in a forest of young lodgepole pine (Table 2). From these figures on gross and net rainfall (X and Y) we calculated an analysis of covariance (Table 3), following the form set up in Table 1. Thence we obtained a linear regression equation which expressed the sampled relation of net to gross rainfall and from which, by inserting individual storm values for gross rainfall, we could compute the most probable values for net rainfall per storm on the whole study area. These calculated values (E), as shown in Table 2, are obviously more stable in their relation to gross rainfall than are the paired sample values for net rainfall, and provide a more reliable estimate of the true net rainfall per storm.

TABLE 3

NET RAINFALL IN A FOREST OF YOUNG LODGEPOLE PINE: ANALYSIS OF VARIANCE AND COVARIANCE

Source of variation	*Degrees of freedom*	*Corrected squares and products*			r*	*Errors of estimate*		
		x^2	xy	y^2		*Degrees of freedom*	*Sum of squares*	*Mean square*
(1) Total	39	3.7162	2.9795	2.6706		38	0.2818	0.0074
(2) Time units (storms)	19	3.7162	2.9795	2.5539	+0.967	18	0.1651	0.0092‡
(3) Observations within storms	20	0.0000	0.0000	0.1167	0.000†	20	0.1167	0.0058

Regression equation: $E = \bar{y} + b(X - \bar{x})$
$= 0.2410 + (0.801760)(X - 0.3160)$
$= 0.8018X - 0.0124$

* r = coefficient of correlation
† No regression available
‡ Not significant; $p > 0.05$

Using equation XI and the data in Table 3, we can estimate the sampling error variance of each of these calculated values:

$$V_E = \left[2s_{\eta'}^2\left\{\frac{20-1}{20}\right\} + s_\xi^2\right]\left[\frac{1}{40} + \frac{(X - 0.3160)^2}{3.7162}\right].$$

In this example

$$s_t^2 = 0.0058 \qquad 2s_v^2 = 0.0092 - 0.0058 = 0.0034.$$

Hence

$$V_E = \left[\frac{(0.0034)(19)}{20} + 0.0058\right]\left[\frac{1}{40} + \frac{(X - 0.3160)^2}{3.7162}\right]$$

which gives

$$V_E = 0.000226 + 0.002430(X - 0.3160)^2.$$

For the first storm measured, with a calculated average net rainfall of 0.060 inch and a gross rainfall of 0.090 inch, this equation becomes

$$V_E = 0.000226 + 0.002430(0.090 - 0.3160)^2 = 0.000350,$$

whence the standard error of the calculated average net rainfall for this storm is estimated to be

$$S_E = \pm 0.000350 = \pm 0.019 \text{ inch.}$$

Thus it can be seen that, by the use of only three instruments, we not only obtained a reliable average value for the whole study period with considerable efficiency, but also derived an average value for each short time unit in the period without much additional work and with relatively high precision. In sampling other variables or even net rainfall in other places, of course, the relative precision of this short-cut method will depend on the degree of correlation existing between observations taken at the fixed and moving stations.

SUMMARY

In the study of microclimate, the investigator may desire to obtain quantitative estimates of the average magnitude of some climatic factor such as rainfall or temperature on one or more areas of land. This objective has been attained in the past by establishing "stations" consisting of rain-gages or other instruments in each area and obtaining continuous records of the desired factor over a period of time. If enough instruments are installed in each area by some scheme of randomization, this method should give satisfactory results; but it may involve excessive costs for the necessary instruments and their care.

The cost of such sampling may frequently be reduced with little loss in precision by the use of a more efficient sampling method which is outlined in this paper. Instead of the considerable number of instruments which is ordinarily required for adequate sampling, this method

calls for only a small number of instruments—two or more—for each area to be sampled. If, for example, it is desired only to obtain an unbiased average value for the area over a period of time, two instruments may be employed. These are placed at randomized positions in the area, and moved after each of a series of short time units (as days or storms) making up the whole period. By this means variations associated with both position and time are sampled simultaneously, so that at the end of the period of study all time variations have been enumerated and a number of positions in the area have been sampled with a comparatively small amount of work.

While this sampling method provides relatively precise average values for each area and period, the resulting average values for the short time units are not so precise because each is based on the sampling of only two locations. If, however, one additional instrument (or more, if desired) is maintained continuously at a fixed position in each area for the whole study period, the records obtained from this instrument and the moving ones can together be employed for the calculation of more reliable average values for each short time unit.

With appropriate modifications, it should be possible to adapt these methods to the sampling of a variety of biological factors in addition to those associated with microclimate.

APPLICATION OF MACHINES TO DIFFERENCING OF TABLES

JACK LADERMAN, *Chemical Warfare Service, War Dept.*
AND MILTON ABRAMOWITZ, *Mathematical Tables Project, U. S. Bureau of Standards*

DURING the past twenty-five years, a variety of machines have been constructed for computational work required in business. Most of these machines, although designed primarily to meet the demands of business, can be applied to scientific computation. Mathematicians, statisticians, and other scientists engaged in computing are in most cases limited to the use of these machines. Although machines designed for specialized computational work exist, and no doubt a machine can be constructed for any type of mathematical computation, they are not available to most computers. Thus it is essential that existing business machines be applied to scientific computation. Great progress has been and will be made in the improvement of such machines if they are more universally utilized in scientific work and their inadequacies brought to the attention of the manufacturers.

The greatest application of business machines to scientific computation has occurred in the field of punched card equipment. The flexibility of the I B M punched card equipment has enabled those engaged in mass computational work to adapt this equipment to perform their tasks in a fraction of the time formerly required by other methods. While considerable progress has been achieved in this field, its full potentialities have not been realized. However, new applications of this equipment to scientific computation are being constantly discovered.

There are other business machines, although much more limited in operation in comparison with I B M equipment which, nevertheless, can be applied efficiently to certain computational work. One such machine is the Underwood-Elliott Fisher Sundstrand Accounting Machine, Model D, which can be applied to differencing of mathematical tables. The main purpose of this paper is to explain this particular application which is of great importance to mathematical computers. Differencing a table until a sufficiently high order difference vanishes or fluctuates within predetermined limits depending upon the order of difference, is the most efficient method by which professional computers check the accuracy of mathematical tables. However, there are many applications of a machine which can obtain differences of high order. For instance, such a machine can be used in the inverse operation of constructing a mathematical table from its differences, for comput-

ing moments by progressive summation, for interpolation, for subtabulation, and for numerical integration.

The first attempt to construct a machine to difference mathematical tables was made by Charles Babbage in 1823 who worked more than ten years and spent approximately $80,000 on his Difference-Engine without success. The main purpose of Babbage's machine was to compute tables from differences, but this operation is essentially the same as differencing tables. After a part of this machine was assembled in 1833, Babbage's thoughts became centered on a new idea, that of a more complicated machine which he called an Analytical-Engine. It is unfortunate that Babbage abandoned his Difference-Engine, for, as it turned out, he was unsuccessful in completing either of the two machines. Had he concentrated on the Difference-Engine, it is very likely he would have succeeded. Nevertheless, Babbage's attempt to build these two machines was not entirely wasted. There is no doubt that Babbage had a considerable influence on improving the precision of machine parts of his time and his work apparently influenced George and Edward Scheutz who succeeded in constructing during Babbage's lifetime a machine to calculate and print mathematical tables from differences.

Since Babbage's time many other attempts were made to construct a Difference-Engine but the machines were impracticable since they were too costly and too limited in their application to other computational work. Recently machines used in general computational work in business have been applied successfully to computing and differencing mathematical tables. The I B M punched card equipment has been used to difference tables[1] and to construct tables from differences.[2] However, only second order differences can be obtained in one run of the cards. To obtain higher order differences, the lower order differences are summary punched on a new set of cards which are run through the machine, their differences summary punched, etc. Another machine which has been used to difference tables is the National Accounting Machine.[3] This machine has been used by Comrie to obtain and print all the differences up to the 5th order. The Burroughs Accounting Machine has also been used as a differencing machine by Comrie in England and by Eckert in the United States. Ten-key adding machines

[1] Hartkemeier, H. P. and Miller, H. E., "Obtaining Differences from Punched Cards," *Jour. Amer. Stat. Assoc.* Vol. 37, p. 285–287, June 1942.

[2] McPherson, J. C., "Mathematical Operations with Punched Cards," *Jour. Amer. Stat. Assoc.*, Vol. 37, p. 275–281, June 1942.

[3] Comrie, L. J., "Inverse Interpolation and Scientific Applications of the National Accounting Machine," *Royal Stat. Soc. Supp.* Vol. III, Nov. 2, 1936.

have been used extensively during the past ten years by the Mathematical Tables Project to compute the nth differences of a table by adding $n+1$ successive values multiplied by the appropriate binomial coefficients. For example, the fifth difference is given by the relation:

$$-\Delta^5 f(x) = f(x) - 5f(x+h) + 10f(x+2h) - 10f(x+3h) + 5f(x+4h) - f(x+5h).$$

In the event that a computer does not possess an adding machine he will find that this differencing process can be performed with considerable success on any of the standard calculating machines such as Marchant, Friden, or Monroe.

Through the efforts of the United States Hydrographic Office, the mathematicians of the Mathematical Tables Project, U. S. Bureau of Standards, and of the Underwood-Elliott Fisher Co., the Model D machine has been set up to difference and construct mathematical tables. Since January 1945 these machines have been computing and printing all the differences up to the 8th order of 12 digit entries merely by setting the entries on a convenient 10-key keyboard and pressing actuating bars. The argument x, the entry $f(x)$, and the eight differences $\Delta f(x-1h)$, $\Delta^2 f(x-2h)$.... $\Delta^8 f(x-8h)$ are being printed on a single line across a 24″ paper roll at the rate of 120 lines per hour. The appearance of the results of differencing a table of x^5 is as follows:

x	$f(x)$	$\Delta f(x-1)$	$\Delta^2 f(x-2)$	$\Delta^3 f(x-3)$	$\Delta^4 f(x-4)$	$\Delta^5 f(x-5)$
6	7776	4651	2550	1230	480	120
7	16807	9031	4380	1830	600	120
8	32768	15961	6930	2550	720	120
9	59049	26281	10320	3390	840	120
10	100000	40951	14670	4350	960	120
11	161051	61051	20100	5430	1080	120
12	248832	87781	26730	6630	1200	120
13	371293	122461	34680	7950	1320	120
14	537824	166531	44070	9390	1440	120

The machine is a multi-register adding machine containing two add-subtract crossfooters and eight adding registers. The subtractions for differencing are performed in the crossfooters; each result is simultaneously printed and stored in one of the registers as well as being applied to obtain the next higher difference. The machine operates in accordance with a specially designed control plate set for this differencing operation. Other operations, such as constructing a table from its differences, can be performed on this machine by changing the control

plate. The designing and setting of a control plate has to be performed by the company but once a plate is set for a particular operation, it need never be changed except for normal mechanical repairs. Several different plates can be maintained with the machine so that the operations can be changed frequently without delay, the interchanging of two control plates being a simple process.

The application of the Underwood-Elliott Fisher Accounting machine, Model D, to the construction of mathematical tables is exceptionally efficient. If the function and all its differences are of the same sign and if the 7th or lower difference is constant, then the machine will compute and print automatically all the lower order differences and the function values without requiring the continuous presence of an operator. The operator would start the machine by setting up the first line containing the differences and the function value. The machine would then automatically repeat the constant difference, compute and print the lower order differences and function value, repeat the constant difference, etc. This is a considerable accomplishment. For example, one can evaluate automatically certain seventh degree polynomials over a long range of equally spaced values of the variable after setting eight quantities into the machine. The function values would then be computed and printed at a rate of three per minute. If the 6th or lower order difference is constant, the arguments can also be printed automatically. The appearance of the results in tabulating x^5 would be as follows:

x	$\Delta^5 f(x-5)$	$\Delta^4 f(x-4)$	$\Delta^3 f(x-3)$	$\Delta^2 f(x-2)$	$\Delta f(x-1)$	$f(x)$
6	120	480	1230	2550	4651	7776
7	120	600	1830	4380	9031	16807
8	120	720	2550	6930	15961	32768
9	120	840	3390	10320	26281	59049
10	120	960	4350	14670	40951	100000
11	120	1080	5430	20100	61051	161051
12	120	1200	6630	26730	87781	248832
13	120	1320	7950	34680	122461	371293
14	120	1440	9390	44070	166531	537824

The machine will operate entirely automatically for certain functions other than those having all their differences of the same sign, but in most cases when the differences change sign, or when the seventh order difference is not constant, an operator must be present to operate the machine. Function values can be computed and printed by inserting any column of known differences up to the seventh order and by press-

ing various actuating bars. The function values can be obtained at a rate of 120 per hour.

The foregoing Underwood-Elliott Fisher Accounting Machine, a standard business machine, accomplishes more than Babbage expected of his Difference-Engine and represents a marked improvement over other machines used to difference tables. The ease with which the operations of the machine can be changed, its comparative simplicity, easy manipulation, and low cost, greatly enhances its practicality and demonstrates how a machine of this type lends itself to specialized computational work. Unquestionably, many more machines of this general nature exist which appear to be unknown to scientific computers. There is no doubt that the great effort currently expended on computational work could be appreciably reduced by stimulating a wider knowledge of the capacities of existing business machines and by promoting a broader extension of their applications.

NOTE ON SAMPLING PROBABILITIES

HOLBROOK WORKING
Food Research Institute, Stanford University

H. G. ROMIG, reviewing in this *Journal*[1] a recent publication of mine, suggests that "It would have been wise for the author to have extended his remarks as many are confused by the fact that the probability of acceptance for product of $\bar{p}$ quality under a given sampling criterion differs from the probability of acceptance for a single lot of $\bar{p}$ quality. In fact, many have considered them identical."

The fact to which he alludes may be illustrated most simply by considering the question: what is the probability of finding no defective article in a sample of five drawn from a lot containing forty articles? The question may be answered on the basis of an assumption regarding either,

a) the quality of the lot, or

b) the quality of the universe from which the lot came.

If the assumption taken be that the *lot* has a fraction defective $\bar{p}=0.1$, the probability in question is

$$P = \frac{36}{40}\cdot\frac{35}{39}\cdot\frac{34}{38}\cdot\frac{33}{37}\cdot\frac{32}{36} = .5729.$$

If the assumption be that the universe or *process* has a fraction defective $\bar{p}=.10$, the specification of lot size has no bearing on the problem, and the solution is simply

$$P = 0.9^5 = .5905.$$

It is apparent that in this instance the probability corresponding to a specified process quality can be determined more easily than that corresponding to a specified lot quality. When the problem presented is less simple, as in the case of single sampling with an acceptance number $c>0$, or in problems of double or multiple sampling, the advantage of working from process quality is relatively greater.

Perhaps the ease with which assumptions regarding process quality can be justified on the ground of simplification has encouraged neglect of the question of principle. Another contributing circumstance has been that when such questions relate to industrial sampling, it is easy to point out that the sampler is usually concerned with a series of samples from an industrial process, and so apparently to justify considera-

[1] September, 1945, pp. 413–15.

tion of process average, ignoring the question whether the industrial process is truly a single process from the probability standpoint.

In any case, it seems worth while to emphasize that the best reason for founding probabilities of industrial sampling on process quality rather than on lot quality is neither a reason of expediency nor one arising from peculiarities of industrial sampling. Such reasons are worthy of notice, but they ought not to obscure the fact that consideration of process quality is to be preferred on grounds of principle. The principle, whether the question be one of the fluctuations of quality in industrial samples or of the sampling fluctuations of an average or of any other statistic or parameter, is that by founding the probability analysis on characteristics of the universe, serious difficulties of inverse probability are avoided.

FREDERICK L. HOFFMAN
1865–1946

FREDERICK L. HOFFMAN who died in San Diego, California, on February 23d at the age of 80 was almost the oldest among the veterans of the American Statistical Association. His earliest book printed as a monograph by the Association appeared fifty years ago and fifteen years later he was chosen for its President. For more than twenty years, 1896–1919, he was one of the most frequent and valued contributors to its publications.

Born and trained in Germany but coming to the United States before he was twenty he brought with him an industry, which more than Teutonic was almost titanic, an eager desire to improve man's estate especially by reducing its burden of sickness, poverty and crime, an unshakable conviction that if one was to do good rather than harm in such an effort one must study the successes and failures of the past and gather widely all relevant facts about the present situation, and that to this end one must rest heavily and constantly upon the statistical method. Becoming statistician of the Prudential Insurance Company of America about ten years after he landed and before he was thirty years old he quickly assembled in his office what was probably the largest collection in the country not only of Federal reports but also of the scattered and fragmentary state and municipal reports dealing with demography in the broadest sense of the term and began to pour out articles and books interpreting this material.

Upon the interpretation of what he gathered, for example whether the greater prevalence of tuberculosis among Negroes is due mainly to inheritance or to environment, or whether the increase in the deaths reported as due to cancer means that cancer mortality is really increasing, opinions might and did differ. His great service lay in focussing the attention of the public and of scholars upon many social evils and furnishing a basis of fact much needed for their successful analysis. This he did for mining and industrial accidents, and for many diseases especially prevalent in certain occupations or of importance because of the large number they enfeeble or kill. These studies were on tuberculosis, malaria and especially cancer.

Thirty-three years ago he read before a medical society a plea for a new organization to study and try to prevent cancer. As a result the American Cancer Society was formed almost at once with him as one of the founders and in his later years he gave most of his attention and devoted most of his publications to that field.

In the middle of the twenty year period which covered his contributions to the Association's Publications he read before it a paper on "Problems of Social Statistics and Social Research" which epitomized his life work before and after. He conceived social research as aiming at "the solution of the problem of poverty with all its resulting problems" and then outlined "a working plan of social research." This was arranged under ten heads: wage-earners' expenditures or budgets, periodical savings aiming to determine whether the prosperity of the masses is or is not increasing with a subdivision of this topic to determine how far the deposits in savings-banks came from wage-earners, safety of investment made by wage-earners, preventable industrial diseases, preventable industrial accidents, employments for persons physically impaired, morbidity and mortality of wage-earners, suicide, and finally "possible physical deterioration" of city populations and particularly of city children. To this paper he added eight short statistical appendices outlining some of the information he had garnered on these problems.

It would repay some student beginning work in these fields to review the subjects on which Hoffman pioneered in order to discover what progress later scholars have made. This would prepare one admirably to carry on his work.

WALTER F. WILLCOX
Ithaca, New York

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WALTER F. WILLCOX
Ithaca, New York

FORREST RHINEHART IMMER
1899–1946

DR. FORREST R. IMMER, associate director of the Minnesota Agricultural Experiment Station, died at St. Paul, Minn., on February 2, 1946, in his forty-seventh year. In his passing the University of Minnesota lost a capable administrator and distinguished plant breeder, and biological statistics lost one of its most constructive leaders. Dr. Immer is survived by his widow, Myrtle Link Immer, and one daughter, Ruth Ann.

Dr. Immer was born in Spencer, Iowa, on July 18, 1899. While a small boy he moved with his parents, Minnie Maas Immer and Albert Immer, to a farm near Jeffers, Minn. He received his high school diploma at Windom, Minn. in 1917 and served a few months in service during the first World War. Subsequently he completed his education at the University of Minnesota, receiving the B.S. degree in 1924, the M.S. degree in 1925, and the Ph.D. degree in 1927.

His professional advancement at the University of Minnesota was rapid. From 1927 to 1929 he was an instructor in Plant Genetics and in 1929–1930 was assistant plant geneticist. In 1930 he became associate geneticist for the Division of Sugar Plants, United States Department of Agriculture. This position he held until 1935, except for the academic year of 1930–1931 during which he was a fellow of the National Research Council, studying statistics with Dr. R. A. Fisher at the Rothamsted Experiment Station, England, and plant breeding at the Svalof Plant Breeding Station, Sweden. He was made associate professor at the University of Minnesota in 1935, and full professor in 1937. In 1941 he was appointed vice director of the Minnesota Agricultural Experiment Station and a year later was elevated to the important post of associate director.

During 1944, Dr. Immer took time out from his administrative duties for special service in England with the Eighth Air Force. Assigned to the Operations Analysis Section, whose duty it was to analyze bombing operations and improve bombing accuracy, he served during the air war in Europe and received citations from General H. H. Arnold and Lieutenant General J. H. Doolittle for exemplary service.

Upon his return in November 1944 to his position as associate director of the Minnesota Agricultural Experiment Station he plunged immediately into the task of fitting agricultural research to the needs of the postwar period. He was given the important posts of chairman, North Central Regional Directors, Farm Structures Committee; chair-

man, Association of Land Grant Colleges Committee on Farm Structures Legislative Bill; chairman, North Central Regional Directors, Poultry Breeding Committee; and member of the Crops Section of the American Society of Agronomy.

For many years Dr. Immer has been consulting editor in statistics of the American Society of Agronomy. Since 1935, he held the position of adviser in applied statistics in the Minnesota Agricultural Experiment Station.

At the time of his death Dr. Immer was on the Editorial Committee for the *Biometrics Bulletin* of the Biometrics Section of the American Statistical Association. He had been very active in the formation of the Biometrics Section within the American Statistical Association in 1938.

Dr. Immer's research has been largely in plant breeding with special emphasis on statistical analysis of research data. Many graduate students at the University of Minnesota came under his direction and guidance in his course in Advanced Agricultural Statistics. Besides 51 publications, most of them in scientific journals, he is author with Dr. H. K. Hayes of a standard textbook *Methods of Plant Breeding*, which was published in 1942.

Among the societies that honored Dr. Immer with membership are Alpha Zeta, Sigma Xi, Gamma Sigma Delta, American Association for the Advancement of Science (fellow), Genetics Society of America, American Society of Agronomy, American Statistical Association, and the Science Club of the University of Minnesota.

His integrity and unswerving devotion to the highest ideals of science, his kindness and untiring devotion to the service of others were a lasting inspiration to all those who had association with him.

E. L. LeClerg
U. S. Department of Agriculture

CARL SNYDER

An Appreciation

CARL SNYDER, past president of the American Statistical Association, died on February 15, 1946, at Serena, Carpinteria, California, at the age of seventy-six after a long illness. He was born at Cedar Falls, Iowa, of an English mother and an American father. He attended the University of Iowa for two years, but his spirit was too restless to be confined to college routines and he gave himself in succeeding years an extensive and liberal education. At twenty he was the editor of a newspaper at Council Bluffs, Iowa.

He lived abroad in Switzerland and England for a number of years, writing on various subjects, but increasingly in the field of business and finance. From 1917 to 1919 he was a special writer on financial subjects for the New York Tribune, and in 1920 began fifteen years of distinguished service with the Federal Reserve Bank of New York, first as the organizer of the Bank's research department, and later as General Statistician. During this fruitful period he lectured frequently at Columbia University, the New School for Social Research, other Universities, and many economic gatherings.

He was president of the American Statistical Association in 1928 and was chairman of the Social Science and Economic Section of the American Association for the Advancement of Science in 1934. He wrote extensively for the Journal of the Statistical Association and other professional magazines and was the author of the following books which had a substantial circulation.

New Conceptions in Science	—1903
La Nuova Scienza	—1904
American Railways as Investments	—1907
The World Machine	—1907
Business Cycles and Business Measurements	—1927
Capitalism, the Creator	—1940

In any generation and in any field of knowledge the new ideas are the product of a few. Most people accept the fashionable ideas of their day, and their service (not a negligible one) is in distributing these ideas and proliferating them in greater detail. In economics a few years ago everybody wrote about the business cycle and spun out the theories of its causes and its consequences. At another period central banks were the fashion and we were told how our troubles could be cured just by having a central bank. To-day the fashion is compensatory

fiscal policy, which has its high priesthood and its ardent devotees. A few people, however, have the originality and the vigor to break out of traditional molds of thinking, and Carl Snyder was one of these. He was an enlightened non-conformist, and his mind cut through drab theory into the realities. He insisted on digging up more and more facts and in learning his economics directly from the facts more than from the textbooks.

The outstanding discovery that Carl Snyder made when he insistently examined all the old facts and persistently dug up masses of new ones was the impetus of growth in economic life under capitalism. He insisted on using the old-fashioned word "capitalism" instead of calling it "the enterprise system" or some other phrase that ducked the prejudices of politics.

He unearthed long series of figures, all of which consistently proved the law of growth. On ratio charts he plotted the figures with trend lines, fitted by eye or by mathematical formula, whichever did the job better. On the basis of these records of history he denied that the American economic system was decadent or doomed to a downward trend by over-saving or maturity.

In his search to see the whole picture and the real picture he joined to his studies of production studies of distribution and prices and wages and bank credit, trying to put together the composite in a way that made sense. Out of all this arose a new over-all measure of the fluctuations in the volume of trade and a new measure of the general price level as distinguished from wholesale prices. With these instruments there became possible for the first time a coherent quantitative measure of the equation of exchange.

Some of these series of measures were rough. The weights to be attached to different kinds of prices in the general price index or in different kinds of production and trade in the trade index had to be approximations. Dollar figures which were "deflated" to turn them into volume figures were subject to substantial error. So there were many critics who thought that these new discoveries were unprecise and departed from the pathway of exhaustive and critical scholarship, as indeed they did occasionally. But they came nearer to measuring the reality of the whole picture than very precise measures of bits and pieces.

It was from these studies that Carl Snyder developed a firm, even a passionate, belief that the Reserve System by its open market and discount policies could and should exercise a strong influence on business fluctuations. His findings and his convictions were no small influence on Federal Reserve policies under the unusual leadership of Governor

Benjamin Strong. The results showed something of both the truth and limitation in these principles. A new chapter was written in central banking theory and practice.

The best fruits of his thinking Carl Snyder put into his book, *Capitalism the Creator*. It is packed with the evidence he had accumulated. It is sprinkled with flashes of his curious, realistic, and vivid mind. It is withal loosely put together and is vulnerable to the critical barbs of the exact student, but it has in it more penetrating wisdom than tons of conventional economic writing.

One can do no better than conclude with a quotation from Virgil Jordan:

> "The burdens of prosperity and progress are borne by but a few creative spirits, insatiably seeking knowledge, recklessly risking danger, enduring endless labor to bring order and accomplishment out of the indifference of nature and the indolence of man."

W. RANDOLPH BURGESS

New York
April 1, 1946

BOOK REVIEWS

Edited by
OSCAR KRISEN BUROS
Rutgers University

Federal Dimensional Quality Control Primer: A Simplified Method for Applying Statistical Quality Control to Dimensions, Revised Edition. Providence 1, R. I.: Federal Products Corp. (1144 Eddy St.), 1946. Pp. 38. Paper. One to six copies, gratis; seven or more copies, $0.25 per copy. *Two reviews follow:*

REVIEWED BY CASPER GOFFMAN
Industrial Mathematician, Westinghouse Electric Corporation
East Pittsburgh, Pennsylvania

THIS booklet gives an elementary presentation of the method of constructing control charts and of their use in controlling machining operations. Several excellent treatments of this subject already exist, notably the *A.S.T.M. Manual on Presentation of Data* and the American War Standards, Z1.1, Z1.2, and Z1.3. (Reviewed in this JOURNAL 40(231): 377–80 S '45.)

It is the reviewer's opinion that the present treatment would at first sight have greater appeal for the shop supervisor or inspector than most existing presentations. On the other hand, it offers no convincing explanation of the practical meaning of statistical control or of the relationship between control chart limits and specification limits. Indeed, the reader is likely to acquire a false understanding of the latter since the control chart limits on averages of 5 are likened to the 3σ limits of a frequency distribution for individuals. No mention is made of a frequency distribution for averages of 5. The reader is likely to believe—as many users of control charts do until shown otherwise—that a process will make good parts if it is in control and has control chart limits within the specification limits. The practical consequences of this misunderstanding can be drastic.

A short section (pp. 4–8) is devoted to a discussion of the variability inherent in a product and the use of the frequency curve to represent the degree of variability. Most of the remainder of the book is concerned with the control chart for averages and ranges.

Another technique, not a control chart technique, is considered on page 26. This consists of drawing action limits at calculated distances from the tolerance limits. The reviewer feels that this method merits greater attention than has been given to it in the book, especially since it has wide application to machining operations with tool wear.

Pages 32–34 consider a plan for operator participation in quality control.

REVIEW BY LLOYD A. KNOWLER
Associate Professor of Mathematics
State University of Iowa

ALTHOUGH *Dimensional Quality Control Primer* was probably written to aid in the sale of precision measuring instruments, it is worthy of rather careful attention by one interested in the field of quality control. As its name implies, the *Primer* is intended to furnish material suitable for reading by the uninitiated—"in no sense to be a complete treatise on the subject." References to some additional material are given at the end of the booklet.

Only the production or fabrication phase of quality control by statistical methods is discussed. This phase is further limited by a discussion of only the so-called average ($\overline{X}$) and range (R) charts. As might well be expected, but slight reference is made to design and specifications based on the process, and little or no reference to acceptance sampling, or to other types of control charts sometimes used in production or fabrication.

The following points, well known by workers in this area and often made by other writers, are probably worthy of special attention: (*a*) "Statistical Quality Control, in itself, is not a complete panacea for assuring the quality of a product. *Engineering and Management action, based upon the facts obtained from the statistical methods, is also necessary*;" (*b*) Statistical quality control is of value to many other industries such as textile, rubber, and optical where interest may not be focused on the mechanical parts; and (*c*) Statistical quality control is suitable for other than large organizations.

The booklet gives reasons for and states certain advantages secured from the use of dimensional quality control. Suggestions are given for the installation of statistical quality control with particular reference to selection of operation and dimension, the sampling procedure, the construction of $\overline{X}$ and R control charts and the interpretation of the results. An interesting example is used to illustrate these points.

In general the ideas of variation are rather well developed. However, figures 8 and 9 might be a little confusing since the frequency curve representing the entire lot in figure 8 is not the sum of the separate parts; and the 3σ limits and the average in figure 9 do not refer to the data set out on the chart. Again some disagreement might result from what was meant to be an elementary analysis of a car on a highway in figure 10.

A reader might question the wisdom of recording the data to four figures as on the work sheet in figure 11 when the deviation from the mean or a nominal would represent much less work, decrease the error hazards, and make calculations simpler. Also, the advisability of rewriting the largest value and the smallest value in computing the range is questioned. The numerical calculations on this work sheet might be rechecked: $\overline{R} = .0004$ is apparently obtained by dividing .0069 by 15. This discrepancy alters the numerical work and some of the analysis on the following pages.

As with many statistical papers, some new notation and variation from

somewhat standard procedures have been developed. For example, $\overline{\overline{X}}'$ and $\overline{R}'$ denote standard values instead of $\overline{X}'$ and R' as set by the American War Standards; crosses have been used for averages and circles for ranges instead of using the same notation for each chart; broken lines have been used as guide for averages, and vertical ones for ranges—this was defended but probably not too convincingly.

The difficult subject of control limits as related to tolerance or specification limits might very well be rewritten. No test formula is given (one might be implied, but it would be necessary to know theory or method to get it) to determine whether or not individual items meet specifications when the process is in control.

The examples and analyses in this booklet are much better chosen than in the first printing. The booklet is of convenient size, has an attractive appearance, and is easily read. The illustrations, diagrams, and pictures are interesting. The above comments are intended to be taken in a constructive manner. Many of the minor objections could be adjusted in the third printing.

It is recommended that the *Primer* be read by those interested in quality control by statistical methods. It represents a considerable amount of work.

Federal Products Corporation is to be commended for making more readily available, material on this important phase of quality control by statistical methods.

Methods in Climatology. *Victor Conrad* (Research Associate in Climatology, Harvard University). Cambridge, Mass.: Harvard University Press, 1944. Pp. xx, 228. $4.00. (London: Oxford University Press, 1945. 22*s.* 6*d.*)

REVIEW BY H. C. S. THOM
Senior Meteorologist, U. S. Weather Bureau
Statistical Laboratory, Iowa State College

THIS is the first full-length book in English on methods in climatology. It is devoted mainly to the application of statistical methods to meteorology, for climatology in a broad sense is simply statistical meteorology. The book definitely reflects the author's long experience as a meteorologist, and he is to be complimented for pioneering a text in this increasingly important field. The book covers a rather large but perhaps somewhat biased sample of the older climatological methods. Many who have been hoping for a text in applied climatology will be disappointed to find that this text does not fulfill their desires since it discusses no applied problems.

The book is divided into four main parts and follows a logical plan of development of the subject. Part I takes up general methods and is largely a discussion of elementary statistical methods. Meteorological or climatological elements together with their measurement and errors of observation are first discussed briefly. Next there are four chapters on statistical methods applied to these elements, the last two chapters of this series being devoted to

curve fitting and harmonic analysis. Part II covers the characterization of the various features of the meteorological elements. The statistical characteristics of the simple elements—temperature, pressure, and wind—are discussed in the first three chapters of this part making use of tools developed in Part I. Here the book departs slightly from its logical order for consideration of certain combined elements and then returns to complete the treatment of the simple elements, cloudiness and precipitation. Part III is concerned with the general problem of spatial comparisons of meteorological elements. After preliminary discussion of the tendency toward uniformity of records of adjacent stations, the problems of homogeneity of climatological series and the completion of such series are treated. A chapter on correlation and regression follows. This order does not seem entirely logical since correlation and regression are the natural tools for the analysis of comparisons and might better have been introduced at the beginning of this part. Graphical methods of comparison, related problems in anomalies, and the special problems of wind occupy the next three chapters. Part III is completed with very brief chapters on air mass climatology and climatic indices. Part IV discusses the climatography which is the general description of the climate by various modes of representation. There are appendices devoted to the forms of climatic tables, tables required in the text, and a handy table of day numbers.

The main defect of the book is the inadequacy of the discussion of statistical methods. While the author mentions modern statistical methods largely by references, few of these methods have found their way into the text. The distinction between the sample and the population is not made clear, and tests of significance are barely mentioned. The fact that statistics themselves are random variables is not explained and little use is made of standard errors of statistics. The statistician will readily recognize the statistical method as that of twenty or more years ago even though references are made to more recent papers on climatological method.

Specific criticisms of the statistical method are principally those of vagueness, inadequacy, or omission. On page 19 the problem of choosing the size of the class intervals is summarized with "The choice of the size of class intervals is dependent upon the number of observations included in the sample and on the degree of exactness required." This gives the reader no method for making the choice. Again on pages 24–29 median quartiles, and deciles are discussed separately but could better have been treated generally as quantiles which are important in climatology. No mention is made of the standard errors of these statistics and their variability is described by "these methods assume their real value only if the size of the sample is so large that half the number of its items represents a series of many hundreds." On page 34 the "more correct" description of the normal distribution of the footnote might have been incorporated in the text. At the bottom of page 35 *et seq.* the fact that distributions must be normal in order that such an interpretation of the standard deviation may be made is too vaguely ex-

pressed. Many distributions in climatology are not normal and care must be exercised in the use of the standard deviation. Neither tests of goodness-of-fit nor errors of forecasts are discussed in the chapters on curve fitting. The discussion of curve fitting and regression could have been simplified by use of the analysis of variance. An early introduction of the analysis of variance would also have meant a great improvement in all of the statistical discussion. In the chapter on comparison of observational series clearer and more meaningful exposition could have been attained by extensive use of regression theory. On page 152 the non-normality of the distribution of r is not mentioned and the student who has acquired the level of statistical knowledge offered by the rest of the book might get into trouble using the probable error of r. A number of statistical concepts important in climatology such as contingency tables, autocorrelation, the tendency to normality of the distribution of certain statistics, special distributions of climatology, the chi-square test and others, and goodness-of-fit of frequency curves are either not mentioned or inadequately treated.

On the climatological side, the book is definitely continental in scope. References to papers of a theoretical nature are largely to the author's own papers or to those of continental writers. The excellent modern works of the Russians have been ignored. To be sure language difficulties here are great; however, abstracts and translations have been provided for a number of the important Russian contributions. References to American and English theoretical works are also few. Both of these countries have been more active in the development of modern statistical methods than the continental countries and both have consequently applied these more extensively to meteorology. Such concepts as the recurrence interval, Thiessen method of weighting, area-depth curves, and others all typically American and important methods in climatology are omitted. The decision for inclusion or omission, of course, remains the prerogative of the author, but this text only runs to 215 pages, a short book for such a large subject.

The author characterizes his own book well in his conclusion which the reader would do well to read first:

> The reader who runs over the pages of this book is perhaps disappointed in not finding this or that special method. In the first pages, it was shown that the number of derived elements is not limited theoretically; thus completeness is unattainable.
>
> Moreover, it is intended to present only a system of methods which facilitates the step from the qualitative to the quantitative. Many ways are offered for attaining this purpose, and they all should lead not only to an exact and comparable description of the climate but, in the end, to a physical explanation.

The number of elements being unlimited does not mean that the number of methods cannot approach adequate completeness. The second paragraph of the conclusion is a broad claim hardly justified by the contents of the book.

This book is finding and will continue to find considerable use as a source book for the qualified statistician working in meteorology and is highly recommended for this purpose. It is not recommended as a text or handbook of climatological methods.

Cleveland Market Data Handbook, 1945 Edition. *Howard Whipple Green* (Director, Real Property Inventory of Metropolitan Cleveland). Cleveland 15, Ohio: Real Property Inventory of Metropolitan Cleveland (1001 Huron Road), 1945. Pp. 69. $2.50.

REVIEWED BY FRANK STROHKARCK
Research Department, The Curtis Publishing Company
Philadelphia, Pennsylvania

THIS handbook, an annual publication, is well known in its field. As the title suggests, the handbook is prepared for the use of marketing men interested in the Cleveland area.

It is a compilation of statistical data drawn from federal and local sources and presents valuable statistics derived from local sources heretofore not published.

The 69 pages which comprise the handbook are made up of 19 pages of promotional text, 34 pages of charts, 13 pages of tables, and 3 pages of index.

The author, as in past publications, has attempted to show too much information in some of the charts; e.g., the bars of the charts are often divided into four or more components. A few of the curve charts are also confusing because of the detail of data shown.

The handbook will be very useful to those interested in obtaining social and economic statistics concerning the Cleveland area.

Statistische Methoden für Naturwissenschafter, Mediziner, und Ingenieure. [Statistical Methods for Natural Scientists, Medical Research Workers, and Engineers.] *Arthur Linder* (Professor of Applied Mathematical Statistics, University of Geneva; Instructor at the Cantonal Technical University in Zurich). Basel, Switzerland: E. Birkhäuser & Co., 1945. Pp. 150. Cloth, Frs. 18.50; paper, Frs. 15.50.

REVIEWED BY HENRY SCHEFFÉ
Associate Professor of Engineering
University of California at Los Angeles

CONSIDERING the general abundance of scientific books written in German, one is puzzled by the lack heretofore of any expounding the familiar (to us) statistical methods employing the t, F, and χ^2 tables. Since this book—an elementary exposition in German—will be of use to few readers of this journal, we shall omit a detailed summary and a listing of errors,* and instead remark on some aspects which may be of wider interest.

* There are a moderate number.

One of these is the organization of the book, which turns out to be quite successful: It is divided into two parts, the first of which, requiring little mathematical knowledge, is in spirit and content similar to R. A. Fisher's *Statistical Methods for Research Workers*, but covers less ground, while the second offers mathematical derivations, mostly by Fisher's geometrical methods, of all the distribution theory forming the basis of the first part.

In spite of its title there is little in this book aimed directly at engineers. Treatment of acceptance sampling is lacking, that of control charts and curve fitting is highly insufficient. The one example of curve fitting in this book is apt to give the engineer or physicist a poor impression of what statistics has to offer him here: A *linear* graph is fitted for the regression of y on x where x is the velocity at which the brakes are applied in an auto and y is the distance required to stop. The range of data is considerable, x varying from 4 to 25 mi/hr, the variance of y is thus far from constant, yet equal weights are used. Later it is decided by a statistical test that the straight line is satisfactory as a regression curve. The reader with a little knowledge of mechanics will be left skeptical or confused. This might have been used as an example of the powerful procedure in which the form (or plausible forms) of the regression curve is determined from physical considerations, after which the parameters (or decision between forms) are determined by statistical methods. The advisability of considering a transformation to logarithms in physical problems could also have been illustrated here. The moral is that it is dangerous to expound applications in a subject matter field without a good feel for its elements.

An interesting difference in mathematical education may be inferred from the fact that while for the first part, according to the introduction, "a knowledge of the simplest algebraic formulas suffices," single and double subscript and summation notation is freely used there. This would trouble most American students who have had a year of calculus. One also finds it strange that in a language so adaptable to the coinage of new words the same word (Streuung) is used for variance and standard deviation. Actually it is used here with five different meanings: for population and sample values of these two quantities (although the distinction is preserved in the mathematical symbols used) and also for the more general idea of dispersion, as when we say "measures of dispersion" (Streuungsmasse).

Operating characteristics (power functions) of tests, and estimation by confidence intervals are not mentioned, although the author comes very close to the latter in the case of the parameters of the binomial and Poisson distributions, and all the necessary distribution theory for many examples of confidence intervals is at hand in the book. Indeed, except for brief mention of a control chart for the sample values of standard deviations, there is nothing that could not have been written twenty years ago after the appearance of Fisher's famous book. Nevertheless, this is a very attractive little book and its author has done a great service to science: He merits the gratitude not only of all experimental workers who read English with less ease than

German, but of statisticians everywhere who should be happy that these methods have been made more widely available, and may even hope that some who would not otherwise have been attracted may be stimulated to further study and perhaps eventual contributions to statistics.

Time Series Analysis: Smoothing by Stages. *Lewis A. Maverick.* San Antonio, Texas: Paul Anderson Co., 1945. Pp. x, 153. $3.00.

REVIEW BY LEONID HURWICZ
Research Associate, Cowles Commission for Research in Economics
The University of Chicago

THE main objective of this book is to present the mechanics of a method of handling time series. This is accomplished by a general description of the procedures to be used, followed by a number of illustrations taken from the field of economics (San Francisco real estate activity; wheat prices and production; crude petroleum output; production of cotton, pig iron, and Portland cement; Erie Canal freight).

A partial justification of the method is contained in the preface along with some references to the author's earlier contributions.

Consider a time sequence of observations X_t on an economic variable X at time t. The time unit may be a year or say, a quarter of a year. Then, it is (implicitly) assumed by the author that the observed values may be decomposed into (1) the trend (possible including "long waves"), (2) the major cycle, (3) the short cycle, (4) the annual cycle (the seasonal), and (5) the irregular (random?) remainder. Write this as

$$X_t = \sum_{j=1}^{5} F_j(t)$$

where F_j is the j-th component in the above listing. (The alternative formulation in terms of products rather than sums gives rise to analogous procedures.)

Defining

$$G_i(t) = \sum_{j=1}^{i} F_j(t),$$

the method of "smoothing by stages" may be described as that of fitting $G_3(t)$ to X_t, $G_2(t)$ to $G_3(t)$, etc. The G's are called "smoothing lines." $G_3(t)$ ("smoothing line A") eliminates the seasonal fluctuations and the irregular remainder, but retains the two cycles and the trend; $G_2(t)$ ("smoothing line B") is free of the short cycle, but still retains the major cycle and the trend; $G_1(t)$ ("smoothing line M") is supposed to contain the trend only.

Fitting is done by moving averages or freehand. A good deal of space is devoted to advice on the methods of fitting. Once the smoothing lines have been drawn, it is possible to obtain the cyclical and seasonal components by subtraction (or, in the multiplicative case, by division).

The properties of the separate cyclical components can now be studied. Further, a claim is made that such decomposition will help correlation studies—either for various components of a given series or for the same component of several series. This is designed to help find certain causal economic relationships. Finally, extrapolation of the smoothing lines is suggested for forecasting purposes. The author is careful to warn with regard to both correlation and forecasting that mechanical procedures must be supplemented by economic theory and corrected for anticipated departures from past patterns.

The author does not use any of the tools of modern statistical inference. There is no explicit statement of the properties of the universe or precisely what it is that is being estimated or tested. Sampling problems are ignored.

Apparently in ignorance of the work recently done in the field of time series (von Neumann, Koopmans, Haavelmo, Mann, Wald, and many others), the author repeats, without taking sides, the once widespread fallacy of nonapplicability of correlation methods to time series.

There was a time when the economist's distrust of statistical procedures was amply justified. But the well-founded objections which had been raised against mechanical application of least squares regressions to economic data have stimulated the development of new methods which utilize economic theory and follow the principles of statistical inference (cf., e.g., Tjalling Koopmans, "Statistical Estimation of Simultaneous Economic Relations," this JOURNAL, December 1945 and references cited therein).

It may be that some elements of the author's method will turn out to be acceptable, if not optimal, for certain purposes. But a proper appraisal must await rigorous formulation of the model to which the method is to be applied.

An Introduction to Educational Statistics. *C. W. Odell* (Associate Professor of Education, University of Illinois). New York 11: Prentice-Hall, Inc. (70 Fifth Ave.), 1946. Pp. xiii, 269. $3.50. *Two reviews follow:*

REVIEW BY PAUL J. BLOMMERS
Director of University Examinations Service
University Examiner and Registrar
State University of Iowa

PROFESSOR Odell's stated purpose in writing *An Introduction to Educational Statistics* was to provide a textbook suitable for use in a one semester course designed to develop in students of education the ability: (*a*) to select appropriate procedures and measures, (*b*) to compute measures, (*c*) to interpret measures reported by others, (*d*) to associate terms and concepts, and (*e*) to present results.

The content, except in the matter of emphasis, does not differ markedly from that of other statistics textbooks written for students of education and

psychology. Briefly stated, it deals with the common descriptive statistics, with measures of relationship, and with sampling error theory. The relative amount of attention given these rather broad and overlapping divisions corresponds to the order in which they have been named. The treatment is (as it must be in a book of this type) nonmathematical.

The difficulties experienced by those who have undertaken the task of preparing nonmathematical textbooks on statistics are generally recognized and appreciated. Wordiness is almost inevitable, and rare indeed is the book of this type which has escaped without some involved and awkward sentences—not to mention involved and awkward expositions. Professor Odell's book is somewhat less mathematical than most volumes of this type and consequently it is not surprising to find such sentences and expositions occurring in it. To illustrate the point, a few examples are submitted: On page 59 is an explanation of the procedure for computing the arithmetic mean of grouped data by the so-called "short method." The description of the entries in the d column is concluded as follows: "In other words, they show how many intervals above or below, that is, larger or smaller than, the assumed mean class each of the other classes is." On page 127 is the sentence: "In general, if a subtest is uncorrelated with the other subtests composing the whole test, the coefficient of correlation between scores on it and on the whole is $\sqrt{1/n}$, in which n is the number of times as long the whole is as the subtest." And on page 131 in a section on interpreting coefficients of correlation: "by dividing $\sqrt{2-2r}$ into values of the measure of deviation employed .5 less and .5 more than the difference in position concerned, using the results as entries in the first column of such a table of the normal curve as is given in appendix, finding the corresponding areas between pairs of ordinates at the given deviation distances, and finally doubling them, the per cents of cases falling within divisions the positions of which differ by the given deviation distances may be found. *This sounds more difficult than it is.*" (Italics mine.)

Perhaps these sample sentences (it is not possible to cite here examples of lengthier expositions) enhance the contention of Professor Helen M. Walker that it is impracticable to present statistics without resorting to some elementary mathematical concepts and that individuals desirous of studying statistics and lacking a knowledge of these concepts had best take time to learn them first. Or perhaps these sample sentences suggest careless editing. They are not to be construed as typical of the writing of the entire volume. The fact remains, nevertheless, that they occurred with sufficient frequency to cause irritation.

Sampling error theory is discussed in the book's final chapter. The treatment is brief. This is surprising in view of the importance of the theory in educational statistics and in view of the liberal amount of space devoted to topics much less complex.

In postponing consideration of sampling theory to the final chapter, the author follows a pattern set in several other recent books of this type. It

seems unfortunate that at least a portion of this material was not introduced earlier in the volume so that advantage could have been taken of every possible opportunity for applying the theory in a variety of situations. It is also invariably true, when sampling theory is thus postponed, that the student must cope with problems in which sampling is implicit without benefit of a preliminary discussion of the theory. In Professor Odell's book, such problems range from the extremes of smoothing frequency distributions (page 38) to the chi-square test for goodness of fit (page 210). In the latter instance the use of chi-square as a test of the hypothesis that a given set of measures is a random sample from a normally distributed universe is described without benefit of any preliminary discussion of sampling theory. In this connection it should be observed that Professor Odell chooses to ignore the work of R. A. Fisher, and to use the number of classes less one as the number of degrees of freedom despite the fact that both the sample mean and standard deviation were used to obtain the theoretical frequencies. Professor Odell is somewhat inconsistent in this position, however, for later in discussing contingency tables (still without benefit of an introduction to sampling) he uses $(r-1)(c-1)$ as the number of degrees of freedom for chi-square.

When the discussion of sampling theory is finally reached, one learns (p. 250) that the author does not approve "the use of degrees or levels of confidence in stating the significance of results." Instead he prefers to state the number of chances to one that the obtained statistic is within a specified distance of the corresponding hypothetical value. In making such statements, several instances occurred in which the "chances to one" phraseology was improperly applied. To illustrate, on page 244, the discussion of an example is concluded with the statement that the chances are 3.7 to 1 "that there is a real difference in the direction indicated." In this connection it should be observed that Professor Odell advocates the use of the area under only one tail of the distribution in stating "chances." Again his position is inconsistent, however, for in the case of the t test applied to means, he employs the area under both tails.

Attention should be called to the fact that in Professor Odell's opinion most of the measures used in education are reported to the next smaller unit rather than to the nearest unit. Throughout the volume, the limits of the classes of grouped distributions are defined so as to be consistent with this view. It should be noted, however, that this view cannot be reconciled with present day theory of educational measurement.

Some prospective users of this text may find to their liking the author's attempt to "enrich" the students' statistical vocabulary. On page 83, for example, he introduces the need for a measure of *variability*, "also called *variation*, *dispersion*, *deviations*, *departure*, *discrepancy*, *spread*, *scatter*, and *fluctuation*." In the opinion of the reviewer the value of this practice is questionable.

It is impossible within the space limitations of this review to give a com-

plete criticism of this book. A few final objectionable features will be cited briefly: (*a*) The author departs from accepted notational practice and uses to represent standard error (Chap. 14). (*b*) The correlation ratio is said to "summarize the deviations of the data from the curved line that *best fits* [italics mine] the means of the rows or columns . . . " (p. 179). (*c*) The median deviation is said to share many of the merits and weaknesses of the standard deviation because it "is regularly found by multiplying the standard deviation by a constant" (p. 100). (*d*) A scheme for changing ordinary ranks to the "same basis" (i.e., to percentile ranks) is presented as the sole requirement for making ranks comparable (p. 77). (*e*) The averaging of percentile ranks is presented as leading to the "average rank." That this "average rank" may not correspond to the percentile rank of this average is not mentioned (p. 78). (*f*) A procedure for correcting the standard error of the mean for the effect of errors of measurement was presented, and the corrected value used in a test in which the fact that errors of measurement were also involved in the obtained mean was ignored (pp. 238 ff.). (*g*) It is suggested that bias in sampling may be revealed by drawing, "in the same manner, but independently, several samples of equal size," and comparing them (p. 230).

REVIEW BY P. E. VERNON
Lecturer in Psychology, University of Glasgow

FROM many points of view this is an admirable book. The reviewer cannot call to mind any other which contains so complete a collection of statistical formulae, with precise directions as to their applicability. The conciseness and clarity of the definitions are a joy to the reader. As a textbook for beginners, however, there is more room for doubt. Professor Odell explicitly puts it forward as covering a one-semester course, and claims to emphasize "practical computation, use, and understanding, rather than derivations and theoretical considerations." No higher mathematics is required.

Possibly the teachers and students whom Professor Odell has in mind are more highly selected than the reviewer's psychology and education students, the great majority of whom would be confused and put off by the very abstractness, precision and comprehensiveness of his treatment of the subject. Standard books such as Garrett's and Lindquist's are perhaps more verbose and in many respects cover less ground, but they would appear to provide the beginner with a better understanding of the uses of statistical methods. In the reviewer's opinion, therefore, this book should chiefly be employed by the research worker for reference, when he has already gained some general comprehension of statistics from other sources. Professor Odell does, of course, restrict his field to some extent by omitting, for example, analysis of variance and covariance and factor analysis, and by giving only very brief accounts of multiple correlation, small sample formulae, principles of experimental design, test reliability, item analysis, and other topics out-

side the usual run. But surely it is more important for the "consumer" to possess some understanding of just these topics than to know, say, the formulae for the standard errors of thirty statistical measures, or several ways of correcting r, S.E.$_m$, etc., which very possibly should be but seldom are used in everyday practice.

A few more detailed points may be picked out for comment. Although chi-squared is one of the most important, and most widely applicable weapons in the statistician's armoury, it is only introduced, in a limited way, towards the end of the book. Actually the coefficient of mean square contingency is described in an earlier chapter without resorting to chi-squared—a coefficient which has little if any practical value, since product-moment r based on mean class values is greatly preferable. The simplest method for the beginner to calculate product moments from grouped data is probably Godfrey Thomson's, which involves the variance along the diagonal of the correlation table; this is not mentioned. The stock method for biserial r, using p, q and z, would seem considerably easier than the method Odell provides. The quadratic formula for tetrachoric r is given without sufficient warning as to the inaccuracy of its approximations. Cyril Burt's trigonometrical approximation is probably closer, and certainly quicker when graphs are not available.

It would be unfair however to list these criticisms without acknowledging how frequently Professor Odell provides novel and extremely useful variations of standard formulae. The reviewer was interested also in his advocacy of the 10th–90th percentile range as one of the best indices of dispersion, for it turned out to be a particularly valuable measure in vocational work during the war in the British Forces. Several of the statistical tables, again, are rearranged in convenient forms, though they tend on the whole to be too coarse for the practitioner; thus the normal curve table only gives heights, areas, etc. for abscissae from 0.0 to 5.0 by intervals of 0.1. There are good exercises and problems at the end of each chapter, but no answers. The typography is extremely clear, and only two misprints have been noted, namely the omission of cross-lines through sets of 5 tallies on page 24, and the absence of σ in the biserial r formula on page 185.

Fundamentals of the Theory of Statistics: Vol. 1, Elementary Statistics and Applications; Vol. 2, Sampling Statistics and Applications. *James G. Smith* (Professor of Economics) and *Acheson J. Duncan* (Assistant Professor of Economics). (Princeton University.) New York: McGraw-Hill Book Co., Inc., 1944, 1945. Pp. x, 720; xii, 498. $4.00 per volume.

REVIEW BY JOHN WISHART
Reader in Statistics, University of Cambridge

IT HAS been a matter of considerable interest to the reviewer to examine a modern text on statistics as his first academic task after six years' absence on war work. The first thing to note is that these books do not simply

represent volumes 1 and 2 of a connected treatise. They are intended to be self-contained expositions, the first elementary and the second advanced. The books have already been reviewed by W. Edwards Deming (this JOURNAL 40(232): 534–6 D '45) largely from the point of view of the need for an exposition of sampling technique. We are here, however, up against two definitions of the word "sampling." The book does not profess, in this reviewer's judgment, to go very deeply into the technique of sampling methods and surveys, and it may therefore be useful for readers to have described to them the achievements of the books as a combined theoretical text, especially as what is meant by "Sampling Statistics," as opposed to "Elementary Statistics," is really tests of significance, exact so far as they can be made exact, as worked out by mathematicians for finite or "small" samples, compared with the more classical "large sample" theory.

The first book is arranged on more or less orthodox "nonmathematical" lines, the inverted commas being used to express the fact that while there is a great deal of algebra (and also geometrical illustration) in the working out, for example, of partial regression coefficients, and some deep mathematical and logical concepts involved in the study of probability and its application to the confidence interval, the book is concerned with calculating the descriptive "statistics" rather than testing their significance. A very great deal of groundwork is covered, and the book is particularly full on the questions of the gathering and presentation of statistics. The reader coming fresh to the subject will learn a great deal of what statistics is about, although it is quite clear that in these days this will not be enough, and he will therefore be required to go on to the subject matter of the second book. This is implicitly recognized by the numerous forward references to the second book. This raises the question of whether it is intellectually satisfactory to leave so many questions unresolved until a more advanced stage in tuition is reached. To some extent the books fall between the two stools of providing self-contained texts for an elementary and a more advanced set of readers. One illustration of this is that the study of the simpler "statistics," i.e., mean, standard deviation, etc., is followed up by tests of significance and the use of the confidence interval, by means of the "large sample" standard errors of the statistics, whereas from chapter 13 onward there is no more mention of such tests. Even the elementary student should be told how to test the significance of a simple regression or correlation coefficient, by means of its standard error or in other, more up-to-date, ways. There is a formula for the standard error of z (where $r = \tanh z$) on page 320, but it has not been applied in the next chapter.

It might have been better to have planned the whole as one standard text, while indicating at the same time what sections could be omitted on a first reading, or by those who were prepared to take on trust the derivations of mathematical formulae, being content if they knew the basic principles on which tests were based, and how they worked. Living, as the reviewer does, in a country where authors are still rather rigidly controlled in the amount of

paper they can use, it is apparent that such a method would have avoided a good deal of the repetition that has gone into the second book with the commendable object of making it self-contained. The second book does not appear on a first reading to be nearly as good in construction as the first. True, it contains a great deal of sound argument, not usual in the ordinary text, on the inferences that can legitimately be made concerning the population parameters from a knowledge of the sample statistics, and it is clear that the authors have studied this question and the work of Pearson and Neyman very thoroughly. But the book is somewhat uneven as a whole, conveying the impression that questions which have specially interested the authors, and which have doubtless been in consequence worked out in great detail in the laboratory, are reproduced in full, while in others the treatment is somewhat sketchy. Since there is little in the modern theory which is not dealt with, it is surprising to find such a small author index, and to note the absence of a number of eminent names, surely the easiest way of finding what has been left out of the book.

It must have surprised the authors to read in W. Edwards Deming's review that "the second is not a textbook in mathematical statistics." The present reviewer thought on examining it that it was intended to be such a text; the present criticism rather refers to the uneven "weights" attached to the different aspects of the subject. It is doubtful whether a long explanation of the distributions of samples of two by means of an arithmetical model is needed when the full composite distribution of mean and variance is later derived; on the other hand it is somewhat startling to find in chapter 12 that mathematical methods are laid aside and Fisher's distribution of r merely quoted. Somehow one feels that a text should be either the one thing or the other. *Either* these distributions should be quoted, explained and illustrated, giving the same treatment to all, *or* they should all be derived as well. Another example is the goodness of fit distribution. The impression is left on the reviewer's mind that the authors are not yet sufficiently at home in the synthesis, and therefore exposition, of the formidable amount of mathematical theory that goes to make up the present-day theory of statistics. There is no reason, however, why with a good deal of revision in a subsequent edition the book should not prove useful for the class of student for whom it is intended. The opportunity could then be taken to correct a fairly large number of minor errors of one sort or another that have no doubt already been spotted. A few of the more important may be set down here. On pages 125–26 the .95 point has been read from table 8 instead of the .98 point; page 142 (footnote) suggests that 11 degrees of freedom, and not 8, have been taken; table 24 has not been calculated very accurately, and there is at least one major error (.2033 for $p_1 = .105$ should be .2136), also the mean should be p_1 in the footnote to the table; the formula in the heading of table 41 is wrong, while it is doubtful whether table 42 should be used for N as low as 11 except as a rough test—Neyman and Pearson suggested it for N greater than 50. On pages 410–11 the authors are evidently referring to a different table

from table 9, since in the latter the values for $n_1 = 9$ appear, which makes interpolation rather pointless, and incidentally shows up the weakness of the interpolation method advocated. One wonders why Fisher's harmonic method on z, half the natural logarithm of F, cannot be used. The authors have incidentally used the .01 values throughout instead of the .05 values in this illustration.

Lastly, a word about analysis of variance. The authors began promisingly in the first book by illustrating analysis of variance in the simple problems in terms of population parameters. It was therefore somewhat disappointing to find the method dealt with very sketchily in the second book, and treated as a somewhat limited tool for special tests. It is not known why they refer to "a so-called 'analysis of variance'." The name is well established and is used by the authors themselves in both books. The process *is* analysis of variance, and so there need be no doubt about the name employed.

Statistical Methods: Applied to Experiments in Agriculture and Biology, Fourth Edition. *George W. Snedecor.* (Director of the Statistical Laboratory, Iowa State College; Head of the Statistical Section, Iowa Agricultural Experiment Station). Ames, Iowa: The Collegiate Press, Inc., 1946. Pp. xvi, 485. $4.50. *Two reviews follow:*

REVIEW BY D. J. FINNEY
Lecturer in the Design and Analysis of Scientific Experiment
University of Oxford, England

DES PREUVES! sans doute il est bon d'avoir des preuves, mais il est peut-être meilleur de n'en avoir pas,"* said General Greatauk of a famous Penguinian trial. Herein lies the great dilemma for the author of a book on applied statistics: he must avoid mathematical proofs, yet to convince his readers he must justify his procedure logically rather than dogmatically. Professor Snedecor has overcome the difficulty very successfully by careful choice of numerical examples and by empirical construction of the probability distributions required in elementary statistical tests. The nonmathematical student should gain in understanding of the tests very considerably by repeating his sampling experiments. Since its first publication in 1937, this book has been one of the few to combine successfully a sound theoretical basis with an exposition sufficiently clear and detailed for those without statistical experience. The rewritten and greatly improved fourth edition contains 60 pages more than the third and has a slightly smaller but better type; it is a valuable text for the experimentalist in any branch of biology, whether he be a novice or experienced in statistics, and no criticism of detail should be considered as disparaging to the whole.

In chapter 1, the concepts of population and sample parameters, null hypotheses, tests of significance, and fiducial limits are introduced through sampling experiments on a table of random numbers. Applications of the binomial distribution are discussed and a useful table of fiducial limits for binomial sampling is given. Professor Snedecor's insistence on the importance

* Proofs! undoubtedly it is good to have proofs, but it is perhaps better not to have them.

of problems of estimation is a valuable corrective to the absurdity of regarding tests of significance as the sole function of statistical analysis. The presentation of the logic of the fiducial argument, however, seems open to criticism in some places, since it may suggest to the reader that a probability of the population parameter lying between certain limits is being determined. In fact, the probability is that of obtaining a sample equally or more extreme from a population whose parameter is specified. Perhaps the reviewer attaches undue importance to the distinction between these two forms of statement, but he is convinced that the student who learns to appreciate this point will be saved from many future illogical arguments.

Continuous distributions are introduced in chapter 2 and the ideas of chapter 1 are extended to them. Properties of normal distributions are investigated in chapter 3 with the aid of repeated sampling from a typical distribution, and the relationships of sample mean, sample variance, and the statistic t are discussed. In chapter 4, tests of difference between two samples are described. The approximate methods proposed for use when the two samples have unequal variances are likely to be sufficiently good for most occasions in which this difficulty arises, but Fisher and Yates' tables, which make the Fisher-Behrens test so easy to apply despite its logical subtleties, might well have been mentioned. The short chapter on computing methods contains many useful hints for the novice and rightly emphasizes the value of common sense appreciation of data in guarding against gross arithmetical errors.

Linear regression is introduced with a very clear explanation of the meaning of the different components of the total sum of squares, though the vague statement that the standard deviation from the regression line is "an average of the vertical distances from the line" might well be amended. Correlation is discussed in the next chapter and its relationship to regression is emphasized. The author fails to make clear that the tests of significance of departure from zero for the linear regression coefficient and the correlation coefficient are identical, and suggests a distinction between the validity of the two; the test of the correlation coefficient is not biased by selection or restriction of values for one variable, providing that the population regression is linear and the frequency distribution within arrays of the other variable is normal.

Chapter 8 describes "large sample" calculations for normal distributions. In chapter 9, the author returns to the analysis of enumeration data and gives a good account of the use of the χ^2 test for contingency tables. In particular, he describes the procedure for $2 \times 2 \times 2$ tables, a method not usually found in textbooks. Chapters 10 and 11 expound the analysis of variance. After a sampling experiment on the variance ratio distribution, the meaning of variance analysis, in terms of additive components of variance, is considered. Some elementary classes of experimental design are described, though the account of these could with advantage be extended so as to indicate the possibilities of incomplete block and lattice arrangements. Indeed, the reviewer would like to see chapter 11 made still longer in order to give a more comprehensive account of analysis of variance in relation to experimental

design. The statement that "one missing item can be supplied with the aid of a formula" should be given more precise phrasing as it tends to perpetuate the belief that loss of a few plots or animals matters little, since the statistician can always find out what the results would have been! The excellent account of modifications required for disproportionate subclass numbers has been much extended from the previous edition. Covariance analysis is well illustrated by typical examples in chapter 12.

In his account of multiple regression, Professor Snedecor pays more attention to correlation coefficients than is usual; the calculation of the inverse matrix directly from sums of squares and products is generally preferable to similar calculations from correlation coefficients. The subsequent account (chap. 14) of curvilinear regression begins with useful suggestions on how to decide the best form of function, and continues with illustrations of the fitting of polynomial regressions. Since published tables of orthogonal polynomial values are sufficient for most practical needs, their use might have been described rather than Fisher's original more complex procedure for fitting orthogonal polynomials to equally spaced observations; the latter method is now only required for observations of unequal weight, a complication beyond the scope of this book.

The account of individual degrees of freedom (chap. 15) is a useful introduction to the analysis of factorial experiments but scarcely does justice to the power of factorial design as an instrument of research; there is no mention of the possibility of reducing the errors of the more important treatment contrasts by confounding those of lesser interest. It is surprising to find a suggestion that Fisher and his associates have met with difficulty in maintaining the thesis that for each degree of freedom an appropriate sum of squares can be segregated! In chapter 16 further aspects of binomial and Poisson frequency distributions are considered, and transformations of data to a form more suitable for analysis of variance are discussed.

The final chapter summarizes the main principles of the planning and analysis of sample surveys and is an excellent introduction to a most important branch of applied statistics which is being increasingly used to day in many fields of investigation. This chapter will be difficult reading for those making their first acquaintance with statistical methods, and possibly in subsequent editions Professor Snedecor might consider its amplification.

The book contains all the usual tables now customary in textbooks of statistics for biologists, together with several less commonly found, of which the table of confidence intervals for the binomial distribution and the short table of range in normal samples are likely to be the most useful.

The reviewer has noticed the following misprints, most of which are trivial: p. 126, l. 9 from bottom: *for* 242% *read* 2421%; p. 221, l. 16 from bottom: *for* 173 *read* 172; p. 231, l. 7 above table: *for* 60.1 *read* 60.4, *for* 3.55 *read* 3.56, l. 5 above table: *for* 60.1 *read* 60.4; p. 245, l. 2 from bottom: *for* 3 points, (a, b), (a, c), and (b, c) *read* 6 points, (a, b), (b, a), (a, c), (c, a), (b, c), and (c, b); p. 268, l. 26: *for* Allen *read* Allan; p. 290, note to Table 11.22: *for*

67,434 *read* 67.434; p. 295, l. 8: *for* 9.8 *read* 9.9; p. 312, l. 8: *for* $(r-1)(6-1)$ *read* $(4-1)(6-1)$; p. 315, l. 14 from bottom: *for* 8.7 *read* 8.8; p. 316, l. 15: *for* Allen *read* Allan; p. 336, last line of Example 12.10: *for* 15.13 *read* 15.12; p. 337, l. 8: *for* 11.10 *read* 11.11; p. 363, l. 11: *for* 0.7412 *read* 0.7411, l. 10 from bottom: *for* 0.7412 *read* 0.7411; p. 408, l. 2: *for* 12 *read* 12.13, l. 4: *for* 12 *read* 12.10; p. 413: Last two paragraphs refer to an analysis of variance in Table 15.2, but no analysis of variance was given there; p. 427, l. 2 above table: for q read k (*twice*); p. 477, Index: *for* Allen *read* Allan; p. 489, Index under "Variance homogeneity of": *for* 349 *read* 249.

REVIEW BY W. J. YOUDEN
Physical Chemist, Boyce Thompson Institute for Plant Research, Inc. Yonkers, New York

PROFESSOR SNEDECOR'S text on statistical methods is written for people without mathematical background with the purpose of teaching them how to apply statistical methods to the data of biology and agriculture. Mathematical statisticians, who are responsible in the first place for establishing the distributions of various statistics, sometimes feel that it is a dangerous business to turn people loose, equipped with a collection of formulas and with only a very imperfect understanding of the theoretical foundations of the subject matter of statistics. Certainly, scientific literature turns up an annual crop of inept applications of statistical techniques with a high yield of erroneous and even absurd conclusions. The reviewer is convinced that many of these mistakes might be avoided if the experimenter did not so often part company with intimate knowledge of his material when he applies a statistical formula. It seems obvious enough that when the results of a statistical analysis are in conflict with an interpretation based upon long experience and familiarity with similar data it is time to show caution and examine carefully the appropriateness of the statistical method used.

This particular hazard is not peculiar to statistics. Similar dangers exist in the use of chemical techniques. It is impossible for nonchemists to avoid chemical procedures; and since they are not chemists, they sometimes make chemical blunders. It may be hoped that Professor Snedecor's hint to seek expert advice when in doubt will be followed whether it be in the field of statistics, chemistry, or something else.

The first chapter, "Sampling of Attributes," introduces some fundamental notions by extended exercises in sampling from a table of random numbers. These samples of a binomial distribution are used to establish experimentally the frequency distribution for chi-square, and this is in turn compared with the exact distribution as a means of establishing confidence in the distributions derived mathematically. The discussion of the term "unbiased" might have made clear that it is used not only with respect to a sample but also to a statistic which is used to estimate a parameter of the population. There is a misprint in the formula at the foot of page 26 where $(1-r)$ is printed for $(1+r)$. One other misprint was noted on page 183, example 8.27, which gives

the equation for the normal curve. The exponent is misplaced so that it appears to be a part of the upper limit attached to the integral sign.

Great care is taken in the early chapters to keep before the reader just what he is attempting to do. It is continually emphasized that an experimenter has some object in mind when he collects data and that the purpose of the statistician is to set up a clear-cut corresponding hypothesis with an appropriate statistical test. Throughout the book, the illustrative examples as well as the exercises are chosen from research data with citations to original sources. This feature greatly enhances the use of the book by research workers who must dig out the methods without benefit of classroom instruction.

After discussing comparisons between two groups, linear regression, and correlation, there is a lengthy and excellent presentation of the analysis of variance and covariance. The remainder of the text covers multiple and curvilinear regression, individual degrees of freedom, a brief treatment of the binomial and Poisson distributions and concludes with an interesting chapter on the design and analysis of sampling which has been added since the first edition. In all, about 150 pages have been added since the first edition. Certain chapters and parts of chapters are indicated as forming a short introductory course in the elements of statistics. The necessary mathematical tables are scattered throughout the book but may be readily found by consulting an index of tables at the back of the book. It is especially encouraging to find how much attention is given to the vicissitudes of actual data. Such difficulties as unequal numbers of items in the groups compared, missing values from otherwise well-planned experimental data, unequal variances in the groups of data under comparison, and transformations of data to bring them into a normal distribution arise continually in actual data and are often ignored in elementary texts.

In conclusion, the question as to whether it is wise for experimental scientists to make elementary statistical applications without possessing a background in mathematical statistics must be passed upon by the scientists themselves. It is evident that ever larger numbers of scientists do find some familiarity with statistics of increasing value. Professor Snedecor's book has already earned the gratitude of many research workers and the new edition, with its inclusion of recent developments in statistics, should get a cordial reception.

Lectures on Statistical Methods of Inspecting and Controlling Quality: Delivered During a Course of Instruction Arranged by the Department of Munitions for Members of the Technical Staff: Held at the University of Melbourne, August 1944. Commonwealth of Australia, Ministry of Munitions, Quality Control Lectures, Melbourne, 1944. Edited by *W. N. W. Wallace* (Head of Quality Control Department, Directorate of Explosives Supply). Lectures by *M. H. Belz* (Associate Professor of Mathematics, University of Melbourne), *J. Churcher* (Engineer, Ammunition Factory, Footscray), *E. A. Cornish* (Officer in Charge, Section of Mathematical Statistics, Council for Scientific and Industrial Research),

H. S. Dean (Assistant Manager, Ammunition Factory, Finsbury), *C. S. Kneebone* (Quality Control Officer, Explosives Factory, Salisbury), *R. G. Moore* (Works Manager, Ammunition Factory, Footscray), *A. L. Stewart* (Standards Association of Australia), *R. S. Thompson* (Engineer, Ammunition Factory, Footscray), *Major Tittensor* (Army Inspection Service) and *W. N. W. Wallace*. East Melbourne S.2, Australia: Council for Scientific and Industrial Research (314 Albert St.), 1944. Pp. 236. Paper, mimeographed.

REVIEW BY EDWIN G. OLDS
Associate Professor of Mathematics
Carnegie Institute of Technology

EARLY in 1943, Mr. A. L. Stewart of the Standards Association of Australia, and Mr. R. G. Moore, Works Manager of the ammunition factory at Footscray, were sent to America by the Australian Department of Munitions to study the application of Statistical Quality Control. While in this country, they conferred with leaders in the field and visited a number of companies where quality control was in operation. Mr. Stewart attended several of the intensive eight-day courses sponsored by the Office of Production Research and Development of the War Production Board, and Mr. Moore visited at least one such course.

Impressed by our success in using statistical methods to improve the quality and increase the production of war material, they returned home convinced that education in the techniques should be made available to Australian manufacturers. Accordingly, an intensive course of instruction was scheduled at the University of Melbourne, August 9–16, 1944, by the Department of Munitions, for those members of its technical staff responsible for the quality of production. A total of eight-seven men attended the sessions held on the opening day and forty-seven of these remained for the entire course.

The book under review is a mimeographed report of the intensive course of lectures. It is assumed that a stenographic record of the proceedings was made and the result carefully edited. The copy read by this reviewer was struck off from badly-worn stencils, and is illegible in spots, but in so far as it can be read, the book has a surprisingly small number of minor errors. Most of these have been noted on a correction sheet at the end of the book.

The reviewer found the book tremendously interesting, and believes his interest will be shared by a great many of the nearly two thousand men who attended O.P.R.D. Quality Control courses in the United States during the war period. Moore and Stewart left the United States while our course was in an early stage, yet the further developments in the two countries followed surprisingly parallel lines. Students of quality control who have been wishing for a complete record of the O.P.R.D. course will find this book an excellent substitute and, in many instances, will have considerable difficulty in convincing themselves that the lectures they are reading are not the ones to which they themselves listened. Other persons will find the book a valuable introduction to the field.

The course at Melbourne was conducted by five instructors, (including Moore and Stewart) and seven special lecturers. After an opening day, arranged to give executives an overview of statistical quality control and its value as a production tool, the class was taught how to construct control charts for measured characteristics, fraction defective, and defects per unit. Lectures on the normal curve, binomial distribution and Poisson distribution, supplied the theory necessary for control charts, and paved the way for later work on acceptance sampling. Reports by the special lecturers served to give the students views of quality control at work.

A considerable amount of attention was given to the use of Sealy's modified control limits, and his method for controlling a group of machines was presented. The distinction between standard and modified limits was carefully made and factors dictating the choice of the latter were noted.

Following a discussion of the general problem of acceptance sampling by attributes, the students were given a thorough explanation of the Dodge-Romig Tables. In the main, this part of the instruction followed closely the material in the three papers originally published in the *Bell System Technical Journal*,* and later issued in book form,† by John Wiley and Sons. Acceptance sampling by variables also received attention.

In comparison with the O.P.R.D. courses, the Melbourne course seems to have devoted considerably less time to demonstration lectures, and to laboratory work by the students. On the other hand, somewhat more theory was included, and such subjects as Fisher's *t*-test, the estimate of sigma from successive differences, testing the reliability of chemical analyses, and rational sentencing of ammunition by use of 5-defect samples were included. Otherwise, the courses were very similar, relying on much the same reference material and using about the same order of presentation of topics. Even the reports on applications by special lecturers sound very familiar. For example, trouble with 100 per cent inspection at one plant was traced to the viewers (inspectors) who "were doing the job to the best of their ability" but whose average age was "of the order of 75 years."

The copy of the book used for this review bears the stamp of the Australian Legation, Scientific Research Liaison Office, 1910 K Street, N.W., Washington 6, D.C. Information is not available here regarding the supply available for distribution, or the possibility that a printed edition may be in preparation. If obtainable, the book will be fascinating to "alumni" of the O.P.R.D. courses, and should be stimulating reading for anyone who wishes to deepen his understanding of the methods and applications of statistical quality control.

* H. F. Dodge and H. G. Romig, "A Method of Sampling Inspection," *Bell System Technical Journal* 8: 613–31 October 1929. H. F. Dodge and H. G. Romig, "Single Sampling and Double Sampling Inspection Tables," *Bell System Technical Journal* 20: 1–61 January 1941. D. B. Keeling and L. E. Cisne, "Using Double Sampling in a Manufacturing Plant," *Bell System Technical Journal* 21: 37–50, June 1942.

† H. F. Dodge and H. G. Romig, *Sampling Inspection Tables*, New York: John Wiley & Sons, Inc., 1944. Pp. vi, 106. *Reviewed September 1946.*

PUBLICATIONS RECEIVED

Statistical methodology publications received are listed elsewhere in this issue under the section "Statistical Methodology Index."

Interviewing for NORC. Denver 10, Colo.: National Opinion Research Center, University of Denver, 1945. Pp. ix, 154. $2.00.

Revenues and Expenses of Newspaper Publishers in 1941: Together With Data Relating to National Advertising in Newspapers. *Neil H. Borden, Malcolm D. Taylor,* and *Richard T. Hovde.* Boston, Mass.: Division of Research, Harvard Business School, 1946. Pp. 56. $0.50. Paper.

Georgia Facts in Figures: A Source Book. By the *Citizens Fact-Finding Movement of Georgia.* Athens, Ga.: University of Georgia Press, 1946. $2.00. Paper.

Jobs and Markets: How to Prevent Inflation and Depression in the Transition. *Melvin G. DeChazeau, Albert G. Hart, Gardiner C. Means, Howard B. Myers, Herbert Stein,* and *Theodore O. Yntema.* Committee for Economic Development Research Study. New York: McGraw-Hill Book Co., Inc., 1946. Pp. xi, 143. $1.60.

Proceedings of the Institute of Economics and Finance, Occidental College, Los Angeles: Fourth Session, February 28, March 1, 2, 1945. Edited by *Cecil L. Dunn* and *Raymond G. McKelvey.* Los Angeles 41, Calif.: Institute of Economics and Finance, Occidental College, 1945. Pp. 161. Paper.

Income From Independent Professional Practice. *Milton Friedman* and *Simon Kuznets.* Publications of the National Bureau of Economic Research, Inc., No. 45. New York 23: National Bureau of Economic Research, Inc. (1819 Broadway), 1945. Pp. xxxiii, 594. $4.50. *To be reviewed.*

Should State Unemployment Insurance Be Federalized? *Herman Gray.* New York 17: American Enterprise Association, Inc. (4 East 41st St.), 1946. Pp. 71. $0.50. Paper.

Detention and Prosecution of Children: Jail Detention and Criminal Prosecution of Children of Juvenile Court Age in Cook County, 1938–1942. *Fred Gross.* Chicago 5, Ill.: Central Howard Association, 1946. Pp. xix, 177. $1.50. Paper.

Postwar Taxation and Economic Progress. *Harold M. Groves.* Committee for Economic Development Research Study. New York: McGraw-Hill Book Co., Inc., 1946. Pp. xiv, 432. $4.50.

Mathematics of Investment, Third Edition. *William L. Hart.* Bound with *Tables for Mathematics of Investment,* Third Edition. Boston, Mass.: D. C. Heath & Co., 1946. Pp. vii, 304; ii, 126. $3.60. *To be reviewed.*

Depreciation Policy and Postwar Expansion. *Lewis H. Kimmel.* Washington: Brookings Institution, 1946. Pp. vi, 66. $0.50. Paper.

The Economics of Housing: As Presented by Economists, Appraisers, and Other Evaluating Groups. *Laura M. Kingsbury.* New York 27: King's Crown Press (2960 Broadway), 1946. Pp. xii, 177. $2.50. Paper.

Commercial Policy in the Canadian Economy. *Orville John McDiarmid.* Cambridge: Harvard University Press, 1946. Pp. xii, 397. $4.50.

The Fundamentals of Accounting: A Cost and Revenue Approach. *Donald H. MacKenzie.* New York: Macmillan Co., 1946. Pp. xiii, 683. $4.00.

The Industrialization of Backward Areas. *K. Mandelbaum.* Oxford University, Institute of Statistics, Monograph No. 2. Oxford, England: Basil Blackwell, 1945. Pp. viii, 111. 10*s.* 6*d.*

Studies in the Classical Theories of Money. *Karl H. Niebyl.* New York: Columbia University Press, 1946. Pp. xiv, 190. $2.50.

In Search of the Regional Balance of America. Edited by *Howard W. Odum* and *Katharine Jocher.* Chapel Hill, N. C.: University of North Carolina Press, 1945. Pp. vii, 162. $3.00.

Small and Big Business: Economic Problems of the Size of Firms. *Joseph Steindl.* Oxford University, Institute of Statistics, Monograph No. 1. Oxford, England: Basil Blackwell, 1945. Pp. v, 66. Paper, 5*s.*; cloth, 7*s.* 6*d.*

The Theory of Price. *George J. Stigler.* An enlarged edition of *The Theory of Competitive Price,* 1942. New York: Macmillan Co., 1946. Pp. vii, 340. $3.75.

STATISTICAL METHODOLOGY INDEX, NO. 4

A QUARTERLY GUIDE TO CURRENT LITERATURE

Edited by

OSCAR KRISEN BUROS

Rutgers University

This bibliographic service presents statistical methodology literature (articles, books, theses, and chapters) published in 1945 to date. Anonymous references are listed first; other references follow in alphabetical sequence by first-named authors. Volume number, issue number (in parentheses), pagination, and date of issue are given for each journal reference. Stars indicate books, theses, and pamphlets; asterisks indicate publications seen by the editor of this INDEX. *Authors of papers on statistical methodology are invited to send reprints to Oscar K. Buros, Rutgers University, New Brunswick, New Jersey in order to facilitate prompt and accurate listings. Foreign-language papers should be accompanied by English translations of titles. Information is desired about references omitted.*

Professor P. C. Mahalanobis. *Sci & Cul* 10(10):422–3 Ap '45.* [473

Sample Surveys. *Economist* 150(5352): 445–6 Mr 23 '46.* [474

Society of Quality Control Engineers in Michigan. *Ind Qual Control* 2(5):15–6 Mr '46.* [475

Statistical Methods in Quality Control: 10, Classification of Defects and Quality Rating. American Institute of Electrical Engineers, Subcommittee on Educational Activities (J. Manuele, H. F. Dodge, A. I, Peterson, and R. E. Wareham). *Elec Eng* 65(3):117–9 Mr '46.* [476

Practical Statistics for Practical Physicists. Ralph Hoyt Bacon (Fairchild Camera and Instrument Corp., Jamaica N. Y.). *Am J Physics* 14(2):84–98 Mr-Ap '46.* [477

Analysis of Scores From Smelling Tests. William Dowell Baten (Professor of Mathematics, Michigan State College). *Biometrics B* 2(1):11–4 F '46.* [478

Some Comments on Carnap's Logic of Induction. Gustav Bergmann (Assistant Professor of Philosophy, State University of Iowa). *Philos Sci* 13(1):71–8 Ja '46.* *See* 317. [479

Approximation of Chi-Square by "Probits" and by "Logits." Joseph Berkson (Division of Biometry and Medical Statistics, Mayo Clinic, Rochester, Minn.). *J Am Stat Assn* 41(233):70–4 Mr '46.* [480

Rapid Least-Squares Solution of Polynomials. Abstract. R. T. Birge (Chairman, Department of Physics) and J. W. Weinberg (Instructor in Physics). (University of California, Berkeley.) *Physical R* 68(3–4): 106 Ag 1–15 '45.* [481

Expectancy Tables: A Method of Interpreting Correlation Coefficients. Reign H. Bittner (Owens-Illinois Glass Company, Toledo, Ohio) and Carlton E. Wilder (Captain, A.G.D., Northington General Hospital, A.S.F.). *J Exp Ed* 14(3):245–52 Mr '46.* [482

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